Labour Market Matching, Stock Prices & the Financial Accelerator

PRELIMINARY AND INCOMPLETE

Alex Clymo*

January 20, 2016

Abstract

I introduce financial frictions into the labour market matching model, and study interactions between the two frictions. Financially constrained "experts" fund vacancy posting and trade in the equity of firms, or "matches", raising debt from the rest of the economy. I demonstrate a feedback mechanism between asset and labour markets which amplifies the model's response to exogenous shocks. Shocks which increase expert net worth allow experts to fund more vacancies, raising market tightness and lowering the ease with which firms can hire workers. This increases the value of being an existing firm, causing stock prices to appreciate. Since experts own firm stocks, this increases expert net worth further, amplifying the initial shock in a mechanism akin to the traditional financial accelerator. I derive an arbitrage equation in my model between equity prices and market tightness similar to the standard free entry condition. I show that any matching model which possesses this arbitrage equation, which includes the standard matching model, is able to match 82% of the volatility in market tightness if it is calibrated to match the volatility in asset prices. Finally, I show that sticky wages amplify the financial accelerator by making the stock market more volatile, and that incomplete markets are crucial for generating the necessary volatility in expert net worth.

JEL classification: E24, E32, E44, J64.

Keywords: financial accelerator, unemployment, asset prices.

*Department of Economics, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, Netherlands. Email: *a.j.clymo@uva.nl*. I thank Wouter den Haan for invaluable support, and seminar participants at the LSE for comments. I thank the ESRC and Paul Woolley Centre for financial support. All errors are my own.

1 Introduction

It has long been understood that financial frictions can amplify business cycle models when they interact with asset prices. A shock which raises the net worth of productive agents will increase their asset demand if they are financially constrained, pushing up asset prices and again increasing net worth, in a cycle known as the financial accelerator. In this paper I show that this accelerator naturally emerges when we add financially constrained agents to the Diamond-Mortensen-Pissarides labour search and matching model.

Any theory of the financial accelerator requires a theory of how the economy's assets are priced. In the original Kiyotaki & Moore (1997) model the asset is land, which is in fixed supply. The asset price is used to clear the market, by ensuring demand equals this fixed supply. In a model where assets are not in fixed supply it is harder to price assets. If production of the asset in question is competitive then the asset price must equal the marginal cost of producing the asset. Thus simple RBC models where consumption is convertible one-for-one into capital generate an always unit price of capital, and no accelerator if capital is used as collateral. Adding adjustment costs to the model allows the marginal cost of producing, and hence the price of, capital to vary over the cycle, and hence reintroduces a financial accelerator.

I show that labour market frictions introduce an accelerator in an analogous way. The asset in my model is filled vacancies, or "matches" between a vacancies and workers. These can be thought of as equity in firms, and are traded by "experts", who also provide the funding for vacancy posting. In equilibrium the price of existing matches must equal the marginal cost of producing new matches, which depends on labour market tightness (the ratio of job vacancies to unemployment, $\theta \equiv v/u$) in an intuitive way: When many firms are posting vacancies the probability of filling your vacancy is low, meaning the marginal cost of producing a filled match is high. As an example, consider a positive productivity shock. This raises the value of a filled vacancy increasing vacancy posting, and hence increases labour market tightness. This increases the marginal cost of producing new matches, and hence pushes up the price of existing matches. This increases the net worth of experts, allowing them to fund more vacancies, and the cycle continues.

In other words, I use the assumption of a frictional labour market to create an upwards sloping supply curve for the economy's assets. This means that increases in demand for assets must lead to asset price increases, creating the financial accelerator. The most natural interpretation of the accelerator is as a feedback between the stock market and the labour market. In one direction, changes in stock prices affect expert net worth and hence the funds available for vacancy posting. In the other, changes in vacancy posting feed back into stock prices by changing the marginal cost of producing new firms, via labour market tightness.

I present a model where the experts are firms owned by the representative household, which could be thought of as banks. However, it is not crucial that they are banks to understand the story. The crucial requirement for my story is that whoever it is who benefits from increases in stock prices is financially constrained, and is the same person who provides funds for new vacancies. This sets the scene for the accelerator because increases in stock prices benefit agents who then reinvest that money in creating new vacancies, which further pushes up stock prices. One could instead model the experts as a separate species and get similar results.

My main result is thus demonstrating the existence of a financial accelerator in the search and matching model, which operates through labour market tightness. Secondly, I derive an arbitrage equation in my model between existing matches and vacancies which is identical to the standard free entry condition, with match value replaced with match price. The free entry condition, equating the value of a filled vacancy with the cost of producing one, holds in the standard matching model for both match value and price, since they are identical. I show that it also holds in my model even after the introduction of financial frictions, for match prices but *not* value. This equation links market tightness and the price of a filled match through the marginal cost arguments made above, and implies a tight link in the model between the volatility of tightness and asset prices: for the standard matching elasticity of one half it implies that the volatility of market tightness must be twice the volatility of asset prices in the model. I construct measures of these volatilities, and show that this implies that if my model is calibrated to match the volatility of asset prices, it can explain 82% of the volatility of market tightness in the data. Furthermore, this holds for any model which shares this arbitrage equation (including the standard matching model) and thus implies that, regardless of the source, any model which can match the volatility in asset prices will do equally well at matching the volatility of tightness. This suggests a potential avenue for work in the matching literature, focusing on improving the asset pricing abilities of these models. This result is inspired by the recent work of Winkler (2015), who argues that the key to generating sufficient amplification from financial frictions models is generating sufficient asset price volatility.

I show that wage stickiness interacts with the financial accelerator. Increasing the degree of wage stickiness in the model increases the gap between the volatilities of the models with and without financial frictions, showing that wage stickiness boosts the amplification given by the financial accelerator. Sticky wages make the stock market more volatile, which boosts the financial accelerator since stock prices feed back into expert net worth and hence vacancy posting. Finally, I examine the role played by market incompleteness in my model. My baseline model assumes, as is common in the financial frictions literature, that agents can trade only in a bond which is not contingent on aggregate shocks. I also solve a version of the model where agents trade a contingent bond, and

show that this version does not deliver any amplification relative to the model without financial frictions. This highlights the key role that market incompleteness plays in generating the financial accelerator. The combination of asset values which are state dependent due to price movements and fixed liabilities generates the volatility of net worth required to deliver volatility in the real side of the economy. With complete markets expert liabilities also become state contingent, and I show that can completely undo the financial accelerator. To avoid confusion, in the rest of the paper reference to a model "with financial frictions" refers to the model with financial frictions and incomplete markets, unless otherwise noted.

The remainder of the paper is structured as follows. In section 2 I review related literature. In section 3 I set up the baseline model with incomplete markets, and in section 4 I set up the models without financial frictions and with complete markets. In section 5 I analyse the differences between the models via their key equations, and in section 6 I present both steady state and dynamic analytical results. Section 7 contains numerical results and robustness checks, and section 8 concludes.

2 Related Literature

My paper is related to several broad strands of literature. It builds on the labour market matching models of Diamond, Mortenson & Pissarides, summarised for example in Pissarides (2000). Within this literature it is also related to papers on the ability of the matching model to quantitatively replicate the data, such as Shimer (2005) and Hagedorn & Manovskii (2008). I contribute to this literature by showing that financial frictions can help resolve the Shimer critique, and by demonstrating the key relationship between asset price volatility and volatility in market tightness.

I also build on the large financial frictions literature. Within this literature my work is closest to those papers which emphasise the interplay between asset prices and net worth, such as the early contribution by Kiyotaki & Moore (1997). My contribution is to show that a financial accelerator naturally arises in my model because of the matching market, even without assumptions on varying marginal products of agents, or adjustment costs on capital. Bernanke, Gertler & Gilchrist (1999) provide a model where adjustment costs on capital provide the movements in the price of capital, as well as an extensive review of the early literature.

My paper is not the first to investigate the intersection between financial and labour market frictions. My contribution here is that my paper is the first, to my knowledge, to use the asset price implications of labour market frictions to generate a financial accelerator. By putting financial frictions on the people who own firms, rather than within the firms themselves, firms' stock market prices affect expert net worth, and hence the funds that experts have to reinvest in their firms

for vacancy creation. This feature is absent from the existing literature. Christiano, Trabandt & Walentin (2011) combine match unemployment and financial frictions in a small open economy framework. Mumtaz & Zanetti (2013) add labour market frictions to the Bernanke, Gertler & Gilchrist (1999) framework and discuss how labor market frictions amplify or dampen the response of the model to different shocks. Petrosky-Nadeau (2013) introduces financial frictions into the search and matching framework and notes changes in amplification and propagation as well as effects on wage bargaining positions. Quadrini & Sun (2015) argue that firms can improve their bargaining position versus workers by taking on more debt, and estimate the effect this has on hiring in a structural model. Schoefer (2015) argues that wage stickiness affects hiring by making the net worth of firms more volatile, impacting the resources they can put towards paying hiring costs.

Other papers study the interaction of labour and finance along other dimensions. Favilukis & Lin (2015) show how sticky wages help explain the equity premium (by making profits more volatile and hence equity riskier) and several other asset pricing facts. Petrosky-Nadeau, Kuehn & Zhang (2013) show how an appropriately calibrated matching model endogenously generates rare disasters, and hence again helps explain the equity premium. Caggese & Cunat (2008) study how financially constrained firms choose between hiring workers on fixed-term and permanent contracts. The result that the financial accelerator relies on incomplete markets has been explored in the existing literature, for example by Carlstrom, Fuerst, Ortiz & Paustian (2014) and Dmitriev & Hoddenbagh (2014). Finally, I exploit the tight link in my model between labour market tightness and stock prices. This feature is present in existing matching models with a free entry condition on vacancy creation. Farmer (2012a) and Hall (2014) present evidence on the tight link between unemployment and the stock market, and Farmer (2012b) exploits this link theoretically in a model of multiple equilibria.

3 Model

The model combines elements of Getler & Karadi's (2011) financial frictions model with the standard search and matching model. As in the standard matching model, I abstract from capital, and instead have experts trade in the equity of firms. Firms and workers must match according to a matching technology, and vacancy posting costs must be paid in order to maintain vacancies. Gertler & Karadi (2011) choose to model experts as a separate species from households, but I instead choose to model them as intermediary firms owned by the households. The name "experts" is retained for consistency with the literature. The model features a representative household, which supplies labour and saves using a risk free bond. Experts post vacancies, trade existing matches in a spot market, and borrow using the risk free bond.

Firms in the model are simply matches between vacancies and workers, and do not face any significant optimisation problem. Note that experts own the equity of all the firms in the economy, which enables me to combine the expert and firm sectors and consider experts directly posting vacancies. My financial structure is thus quite stylised. In particular, all firms are funded with equity from experts, and all experts are funded with risk free debt from the household. The household is unable to directly invest in the equity of firms. Time is discrete and the horizon infinite.

3.1 Individual expert's problem

Experts are owned by the representative households. They are restricted severely in their equity issuance: they cannot raise money via equity, and must purchase assets using retained earnings or debt. Experts exit exogenously each period with probability $(1-\sigma)$ and new experts are created each period so that the mass of experts is constant. New experts receive an exogenous equity injection from the representative consumer. If the value of being an expert exceeds one (as it does in a neighbourhood of the steady state) experts will not pay out dividends until they exogenously shut down. In this case, an expert's balance sheet gives us:

$$Q(s)k_o' + \kappa z v = d' + n \tag{1}$$

n is beginning of period net worth, which is a state variable from the expert's point of view, and s is the aggregate state. d' is borrowing, which is combined with net worth to purchase assets. All aggregate variables are indexed by the aggregate state, s. Individual level variables, such as d', will be denoted without reference to state variables before they are optimised, and $d' = d(s, n)$ will refer to their optimised, equilibrium values. On the left hand (asset) side, the expert has two choices. Firstly, she can buy an existing match on the spot market for price $Q(s)$. The number of existing matches she wishes to purchase is denoted by k_o'. These matches produce next period, and then a fraction ρ_x exogenously separate. Those that don't separate can be resold tomorrow for price $Q(s')$. Alternatively, the expert can decide to set up some new matches herself by issuing vacancies, v. She pays a flow vacancy cost κz per vacancy, where z is aggregate productivity, and a fraction $q(\theta(s))$ are successful. q denotes the vacancy filling probability and θ market tightness, both of which individuals take as given. If a vacancy is successful today then it produces for sure tomorrow, and then a fraction ρ_x exogenously separate. Hence notice that buying a existing match today or setting up a match yourself yield the same payoff tomorrow.

I assume away the idiosyncratic risk that a expert's vacancies don't match by assuming that the expert issues a continuum of vacancies. Thus if an expert posts v vacancies today then it gets for sure $q(\theta(s))v$ successful matches, and we can think of the expert as directly choosing the number of successful matches, k_n', as opposed to the number of vacancies. So if we define $k_n' \equiv q(\theta)v$ we can

rewrite the balance sheet as:

$$Q(s)k'_o + \frac{\kappa z}{q(\theta(s))}k'_n = d' + n \tag{2}$$

Note that experts can only post non-negative vacancies, so $v \geq 0$ is a constraint for the expert. This implies the equivalent constraint $k'_n \geq 0$ as long as the probability of a successful match is non zero ($q > 0$). Expert net worth next period is the return on assets less the repayment of debt:

$$n' = \left(z' - w(s') + (1-\rho_x)Q(s')\right)\left(k'_o + k'_n\right) - r(s)d' \tag{3}$$

Where $w(s)$ is the wage, which depends on the aggregate state, and $r(s)$ is the interest rate on debt. Combining this with the balance sheet equation gives:

$$n' = \left(z' - w(s') + (1-\rho_x)Q(s') - r(s)Q(s)\right)k'_o + \left(z' - w(s') + (1-\rho_x)Q(s') - r(s)\frac{\kappa z}{q(\theta)}\right)k'_n + r(s)n \tag{4}$$

I derive a constraint on borrowing using Gertler & Karadi's (2011) limited commitment problem. Within this period but after raising funds, experts can abscond with an amount of resources equal to a fraction Λ of the value of the assets they invested in. The remaining fraction $1-\Lambda$ is exogenously destroyed, leaving nothing for the lender to recover. If experts abscond they lose the franchise value of being an expert, but gain the stolen resources. Since this is a within-period problem, lenders can anticipate exactly when an expert will abscond with their resources and they will restrict the amount they lend to make sure this doesn't happen. Define the value function conditional on a choice of (k'_o, k'_o) as $V^*(n, s; k'_o, k'_n)$. Then this limited commitment problem gives the constraint:

$$\Lambda\left(Q(s)k'_o + \frac{\kappa z}{q(\theta(s))}k'_n\right) \leq V^*(n, s; k'_o, k'_n) \tag{5}$$

This requires that the value of the expert must exceed the value of the assets she has the potential to steal, in order to guarantee that the expert does not have an incentive to abscond with them. The conditional value function is given by:

$$V^*(n, s; k'_o, k'_n) = \mathrm{E}\left[\Omega(s', s)\left((1-\sigma)n' + \sigma V(n', s')\right)\middle|\, s\right] \tag{6}$$

Where $\Omega(s', s) \equiv \beta u'(c(s'))/u'(c(s))$ is the consumer's stochastic discount factor (SDF), and where n' is replaced with the value implied by (4). $V(n', s')$ is the overall maximised value next period, and today's value is given by the maximisation:

$$V(n, s) = \max_{(k'_o, k'_n)} V^*(n, s; k'_o, k'_n) \tag{7}$$

Subject to (4), (5) and $k'_n \geq 0$. The following lemma summarises the solution to the expert's problem. I focus on the case where the non-negativity constraint on vacancies never binds, since my expert sector will aggregate and this case is thus consistent with the observation that total vacancies are always positive in the data.

Lemma 1. *If the non-negative vacancies constraint isn't binding and prices are such that the expert cannot acquire infinite value, then the solution to the individual expert's problem requires that capital price and tightness satisfy:*

$$Q(s) = \frac{\kappa z}{q(\theta(s))} \tag{8}$$

This implies that old and new matches yield the same return, and individual experts are indifferent between the two and optimise over the sum $k' \equiv k'_o + k'_n$. Defining leverage as $\phi \equiv Q(s)k'/n$, expert net worth evolves as:

$$n' = \big(\big(r_k(s', s) - r(s)\big)\,\phi + r(s)\big)\, n \tag{9}$$

Where $r_k(s', s)$ is the return on investing in a match, given by:

$$r_k(s', s) \equiv \frac{z' - w(s') + (1 - \rho_x)Q(s')}{Q(s)} \tag{10}$$

Optimal leverage is independent of expert net worth, and optimal k', d' and V are linear in net worth. Expert value is given by $V(n, s) = \nu(s)n$, where $\nu(s)$ is defined recursively by:

$$\nu(s; \phi) = \mathrm{E}\left[\Omega(s', s)\left(1 - \sigma + \sigma\nu(s')\right)\left(\left(r_k(s', s) - r(s)\right)\phi + r(s)\right)\Big|\, s\right] \tag{11}$$

and $\nu(s) = \max_\phi \nu(s; \phi)$ subject to the moral hazard constraint $\Lambda\phi \leq \nu(s; \phi)$. Equilibrium leverage, $\phi = \phi(s)$, is given by the value that solves that maximisation. Total match demand, $k' = k(s, n)$, and debt, $d' = d(s, n)$, are given by $Q(s)k(s, n) = \phi(s)n$ and $d(s, n) = (\phi(s) - 1)n$. If the moral hazard constraint binds then value and leverage are jointly determined by:

$$\nu(s) = \mathrm{E}\left[\Omega(s', s)\left(1 - \sigma + \sigma\nu(s')\right)\left(\left(r_k(s', s) - r(s)\right)\phi(s) + r(s)\right)\Big|\, s\right] \tag{12}$$

$$\Lambda\phi(s) = \nu(s) \tag{13}$$

If the moral hazard constraint isn't binding then prices must satisfy:

$$\mathrm{E}\left[\Omega(s', s)\left(1 - \sigma + \sigma\nu(s')\right)\left(r_k(s', s) - r(s)\right)\Big|\, s\right] = 0$$

The expert is then indifferent about her leverage, and expert value is given by:

$$\nu(s) = \mathrm{E}\left[\Omega(s', s)\left(1 - \sigma + \sigma\nu(s')\right) r(s)\Big|\, s\right]$$

The proof of this lemma is left to the appendix, but the intuition and results are straightforward. The idea is that as long as an expert wants to post vacancies ($v > 0$) then she must be indifferent between posting vacancies and purchasing existing matches on the stock market. This is because the two assets have identical payoffs tomorrow, and the moral hazard constraint does not restrict the expert from performing arbitrage between them. For the expert to be indifferent it must be

that the cost of purchasing an existing match, $Q(s)$, is equal to the cost of producing a new match, $\kappa z/q\left(\theta(s)\right)$, as stated in equation (8). Notice that this equation is identical to the free entry condition found in the standard matching model (if the value of a match is replaced with the price of a match), even though the derivation is different.

3.2 Aggregating the experts

Experts as a whole enter the period with total undepreciated matches K, and a total value of debt to be repaid D. Define N_c as net worth from continuing experts and N_e as net worth from new experts. Total expert net worth at the beginning of the period is thus:

$$N(s) = N_c(s) + N_e(s)$$

Where each of the components is given by:

$$N_e(s) = (1-\sigma)\, w_e z$$

$$N_c(s) = \sigma\left(\left(z - w(s) + (1-\rho_x)Q(s)\right)K - D\right)$$

New experts get net worth proportional to aggregate productivity. The net worth of continuing experts comes from the output and resale value of their total capital less their debt repayment. Thus overall expert net worth evolves according to:

$$N(s) = \sigma\left(\left(z - w(s) + (1-\rho_x)Q(s)\right)K - D\right) + (1-\sigma)\, w_e z \tag{14}$$

The transitions for K and D can be found by aggregating the individual policy functions:

$$K'(s) = \frac{\phi(s)N(s)}{Q(s)} \tag{15}$$

$$D'(s) = r(s)\left(\phi(s) - 1\right)N(s) \tag{16}$$

Note that the definition of D is slightly different from that of d: D is defined to contain the interest rate, for convenience when it is used as a state.

3.3 The labour market

The structure of my labour market is standard. The total mass of workers within the household is normalised to one, so unemployment at the beginning of the period is given by:

$$u(s) = 1 - K \tag{17}$$

Tightness is defined as usual as the ratio of total vacancies to unemployment:

$$\theta(s) = \frac{v(s)}{u(s)} \tag{18}$$

The matching function is assumed to take the constant returns to scale Cobb Douglas form:

$$m(s) = \psi_0 u(s)^{\psi_1} v(s)^{1-\psi_1} \tag{19}$$

This allows us to express the vacancy filling probability as a function only of tightness:

$$q(s) = \frac{m(s)}{v(s)} = \psi_0 \theta(s)^{-\psi_1} \tag{20}$$

Total employment next period is the sum of new and undepreciated matches:

$$K'(s) = m(s) + (1 - \rho_x)K \tag{21}$$

In this paper I take a reduced form approach to wage determination, rather than modelling the mechanisms more explicitly. While this approach is somewhat unsatisfactory, I note that the focus of the present paper is to understand the links between financial frictions and the matching model. Hence as long as the wage process is chosen carefully to realistically match the data on wages one would expect my results to also hold in a model with alternative wage determination mechanisms which were also able to match wage data. Following Michaillat (2012), I assume that the wage is an exogenous function of productivity:

$$w = \bar{w} z^{\gamma} \tag{22}$$

$\bar{w}$ thus controls the average wage and γ the degree of wage rigidity. $\gamma = 0$ corresponds to wages which are completely rigid, and $\gamma = 1$ corresponds to wages which move one-for-one with productivity. This is the equilibrium outcome of fully flexible Nash-wages in the standard DMP model when vacancy posting costs are proportional to productivity, unemployment income is proportional to wages, and the utility function is logarithmic. Hence I refer to low values of γ as generating relatively rigid wage, and higher values generating flexible wages.

3.4 Goods market and household problem

In the baseline incomplete markets model the household lends to the expert using a risk free, one period bond. Household optimality requires that the interest rate satisfies the standard Euler equation:

$$r(s) = \frac{u'(c(s))}{\beta \mathrm{E}[u'(c(s'))|s]} \tag{23}$$

Where $c(s)$ is consumption. Since labour of each household member is indivisible, there is no labour supply choice. I assume that the household always chooses for all of its unemployed members to

search for a job, and hence the Euler equation above is the only household optimality condition. Goods market clearing requires that all output is either consumed by the household, or used to pay vacancy posting costs:

$$zK = c(s) + \kappa z v(s) \tag{24}$$

Finally, productivity follows a stationary AR(1) process:

$$\log z' = (1-\rho)\log \bar{z} + \rho \log z + \sigma_z \varepsilon' \tag{25}$$

Where ε is an independent and identically distributed standard normal. $\bar{z}$ controls the mean of productivity, ρ its autocorrelation, and σ_z the standard deviation of productivity innovations.

3.5 Definition of equilibrium

The state can be solved with three state variables: $s = (z, K, D)$. Productivity, z, and employment, K, the state variables in the standard matching model, are augmented with the debt repayment made by experts to the household, D. Since I will be linearising around a steady state where the financial friction binds, I define equilibrium under the assumption that the financial friction always binds:

Definition 1. *Incomplete markets equilibrium (IME) is a sequence of quantities and prices v, ϕ, N, D, K, Q, θ, m, q, u, r, c, z, w, r_k, and v such that:*

1. *Households optimise taking prices as given: (23)*
2. *Experts optimise taking prices as given: (8) (10) (12) (13) (14) (15) (16)*
3. *The goods market clears: (24)*
4. *The labour market evolves according to the matching function: (18) (19) (20) (17) (21)*
5. *The wage is given by the wage rule: (22)*
6. *Productivity evolves according to: (25)*

In the next section I set up two comparison models with different financial structures. Competitive equilibrium is defined similarly to the above for these economies, and the definitions are excluded for brevity.

4 Comparison models

4.1 Model without financial frictions

The model without any financial frictions (i.e. no limited-commitment problem, and market completeness) dispenses with most of the expert equations, and instead has matches valued simply using the consumer's SDF. The derivation is standard and is omitted. Given the definition of r_k optimality can be compactly stated as:

$$\mathrm{E}\left[\Omega(s',s)r_k(s',s)\middle|\, s\right] = 1 \tag{26}$$

This is simply another way of writing the usual recursion for job value. This model is simpler than the model with experts because we do not have to worry about financial variables. The definition of equilibrium, which I label a Standard Equilibrium (SE), is similar to the definition of an IME, replacing the expert optimality equations with just (26).

4.2 Complete markets model

I now consider the model where experts and consumers trade state contingent securities instead of just risk free debt, but we retain the limited-commitment problem. As in the incomplete markets model, arbitrage between vacancies and existing matches allows me to combine them into a single asset, which I impose from the start for simplicity of exposition. An individual expert's balance sheet is now:

$$Q(s)k' = n + \int_{s'} d(s')\,\mathrm{d}s' \tag{27}$$

Where $d(s')$ donates the quantity of securities purchased that are payable if next period's state is s', and $\mathbf{d} \equiv \{d(s')\}$ denotes the collection. Net worth evolves according to:

$$n' = \left(z' - w(s') + (1-\rho_x)Q(s')\right)k' - r(s')d(s') \tag{28}$$

The interest rate is state contigent and derived from the representative household's optimality:

$$r(s') = \frac{u'(c(s))}{\beta u'(c(s'))p(s'|s)} \tag{29}$$

Where $p(s'|s)$ is the marginal density of the state s' conditional on today's state, s. Expert value conditional on a choice of k' and $\mathbf{d}$ is given by:

$$V^*(n,s;k',\mathbf{d}) = \mathrm{E}\left[\Omega(s',s)\left((1-\sigma)n' + \sigma V(n',s')\right)\middle|\, s\right] \tag{30}$$

Where it is understood that n' is replaced using (28). Experts maximise overall value:

$$V(n,s) = \max_{k',\mathbf{d}} V^*(n,s;k',\mathbf{d}) \tag{31}$$

Subject to (27), (28), and the moral hazard constraint:

$$\Lambda Q(s)k' \leq V^*(n, s; k', \mathbf{d}) \tag{32}$$

Note that now the maximisation is also over all the state contingent securities, **d**. The following proposition establishes the central result of the complete markets model:

Lemma 2. *Assuming that the moral hazard constraint is always binding, there is a solution to the individual expert's problem in the complete markets model featuring constant leverage, $\bar{\phi}$. Expert value is linear in net worth, with a constant first derivative: $V(n, s) = \bar{\nu} n$. Leverage satisfies $\Lambda\bar{\phi} = \bar{\nu}$, and the solution requires that:*

$$\mathrm{E}\left[\Omega(s', s) r_k(s', s) \middle| s\right] = \frac{\bar{\nu} - (1 - \bar{\phi})(1 - \sigma + \sigma\bar{\nu})}{(1 - \sigma + \sigma\bar{\nu})\,\bar{\phi}} \tag{33}$$

The proof is relegated to the appendix, but the intuition for constant leverage and marginal value is relatively simple. With complete markets the expert is able to use contingent debt to allocate resources across future states of the world. She actually has a lot of freedom to do this, because the moral hazard constraint limits her overall borrowing, not how she allocates debt across the contingent states. This is why marginal value, $\bar{\nu}$ has to be constant, because if it was not the expert would use contingent debt to borrow in states with low value, and transfer those resources to states with higher value. Since she is unconstrained in doing this and value is linear in net worth, she would take infinitely large positions, and achieve infinite overall value. This would violate that the moral hazard constraint is binding (which I assumed) since with infinite value she can always borrow more. Given constant marginal value, constant leverage follows trivially from the binding borrowing constraint.

Given the constant marginal value and leverage, equation (33) delivers the main result of the complete markets model. Notice that the right hand side is constant, and that apart from this the equation is identical to the first order condition of the standard equilibrium, (26). In the limiting case of $\bar{\nu} = 1$ the two equations are exactly identical, and the standard and complete markets models deliver identical equilibria. In general, the two models are identical up to this "wedge" due to the moral hazard constraint, and we will see in later sections that they deliver very similar dynamics.

5 Discussion of key equations

Having set up the three models, in this section I discuss the differences between them using two key model equations: the free entry or arbitrage equation, and the discounted sum pricing matches. This thus serves as an introduction to the models before moving on to more explicit analytical and numerical results in later sections. One key idea from this section is the tight link that the matching

model imposes between the volatility of market tightness and asset prices, which is an idea also taken up in Hall (2014).

5.1 Comparing equations: The free entry condition

The first thing to note is that both the financial frictions models and the standard matching model contain the familiar equation:

$$Q(s) = \frac{\kappa z}{q\left(\theta(s)\right)} = \frac{\kappa z}{\psi_0}\theta(s)^{\psi_1}$$

The interpretation is slightly different in the three models. In the standard matching model this is the free entry condition, stating that the value of posting a vacancy should be equal to zero. Q, the value of a filled vacancy, should be equal to the cost of posting a vacancy, adjusted for the probability of success, leaving no surplus. Notice that in the standard matching model Q is both the value of a filled vancancy, and the price a filled vacancy would trade on the market.

However, in the financial frictions models we must be careful because Q must be interpreted as the market price of a filled match. This will be different from the value of a match to an expert due to the financial friction. In the standard model the value of posting a vacancy must be equal to zero in equilibrium, but this is not true if the financial friction binds: Experts would like to post another vacancy, they just don't have the funds. If this is the case, how do we still recover an equation identical to the free entry condition of the standard model?

This is because the equation is in fact a no-arbitrage equation for the experts, which says that they must be indifferent between creating a new match themselves (by posting $1/q$ vacancies) and purchasing an existing match on the spot market. Because I have assumed away vacancy risk, these two choices represent assets which give identical payoffs in the future: if you buy a match today or create one, you have a match tomorrow in either case. Hence if they had different prices experts could make infinite profit by going long in one and short in the other, which cannot happen in equilibrium. This is not prevented by the financial friction. In other words, matches are priced at marginal cost, as we would expect as the spot market for matches is competitive.

A complete proof can be found in the appendix, and it relies on aggregate vacancy posting being positive, which I assume. In the appendix I also provide another way of deriving the free entry condition in my model, by assuming the existence of competitive "match producing firms" who create matches and sell them to experts.

Another way of putting this is to note that this equation is the key to understanding where stock market value derives from in this model. Since there is no physical capital in the economy, matches between workers and firms are the only physical asset which can be owned. But these matches only have value to the extent that they can't be replicated costlessly: as discussed above, they are priced

at marginal cost. In the limit where matching frictions disappear ($\kappa \to 0$), equation (8) implies that $Q = 0$ at all times. This is because current matches must be a worthless asset in order for this equation to hold, since they can be costlessly replicated by posting enough (free) vacancies.

At this point it is worth noting that the free entry or arbitrage conditions place a very strong link in the model between the volatilities of asset prices and market tightness. To see this, take logarithms of (8) to get:

$$\log \frac{Q(s)}{z} = \log \frac{\kappa}{\psi_0} + \psi_1 \log \theta(s)$$

This implies a very strong link between the standard deviations of labour market tightness and asset prices:

$$\sigma\left(\log \theta\right) = \frac{1}{\psi_1} \sigma \left(\log \frac{Q}{z}\right) \tag{34}$$

Where $\sigma(x)$ refers to the standard deviation of variable x. This implies that the volatility of log tightness, a key moment in the search and matching literature, is pinned down exactly by the volatility of log asset prices (scaled by labour productivity). Given the standard value of $\psi_1 = 1/2$ the above equation implies that the model will always generate a volatility of tightness twice that of asset prices. To the extent that the introduction of financial frictions can increase the volatility of asset prices we should thus expect them to increase the volatility of the labour market as well via this arbitrage equation.

5.2 Comparing equations: The discounted sum

Since the models with and without financial frictions both have the same free entry condition, where is the substantive difference between them? In this section I show that much of the difference between the models can be understood via the recursion for the price of a match. In the standard matching model the value of a filled match to its owner can be expressed recursively as:

$$Q(s) = \mathrm{E}\left[\Omega(s', s)\left(z' - w(s') + (1 - \rho_x) Q(s')\right)\middle| s\right]$$

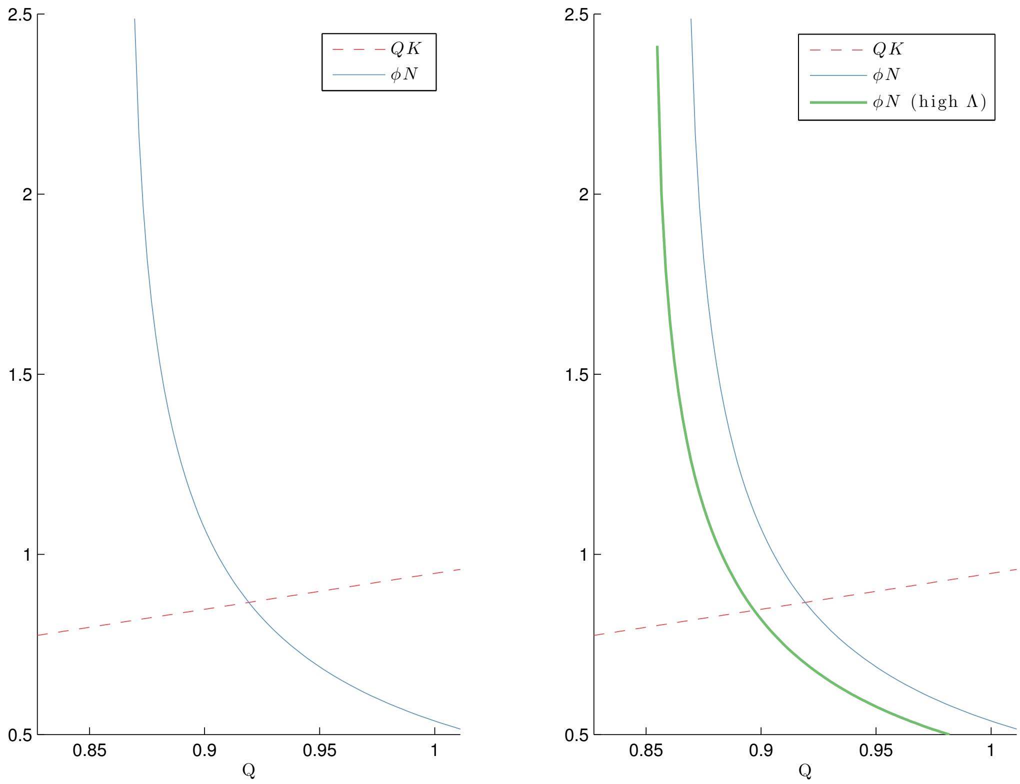
As previously explained, this is also equal to the market price of a filled match since there are no financial frictions. However, in the appendix I derive the following expression for the price of a match in the model with financial frictions and incomplete markets:

$$Q(s) = \frac{\mathrm{E}\left[\tilde{\Omega}(s', s)\left(z' - w(s') + (1 - \rho_x) Q(s')\right)\middle| s\right]}{\mathrm{E}\left[\tilde{\Omega}(s', s)\middle| s\right] / \mathrm{E}\left[\Omega(s', s)\middle| s\right] + \frac{\lambda(s)\Lambda}{1 + \lambda(s)}}$$

Where $\tilde{\Omega}(s', s) \equiv \Omega(s', s)\left(1 - \sigma + \sigma \nu(s')\right)$ is the expert's SDF, which is the household's SDF "twisted" by the fact that, due to the financial frictions, the value of funds might be higher inside the intermediary than in the hands of the household . The two recursions are similar except that in the

incomplete markets recursion: 1) the numerator uses the expert's SDF instead of the consumer's and 2) the recursion is divided by some extra terms.[1] $\lambda(s)$ is the equilibrium lagrange multiplier on an expert's limited commitment constraint. The equation shows us that, ceteris paribus, $Q(s)$ is decreasing in $\lambda(s)$. In other words, the more the financial constraint binds, the lower is the asset price, which is intuitive since we are discussing asset demand, and a tighter borrowing constraint reduces the funds available to purchase assets, pushing down prices. The arbitrage equation, (8) reveals that a lower price must also mean lower vacancy posting. This means that we can tell a

Figure 1: Graphical solution to the steady state of the financial frictions model



The left panel plots $L(Q)$ in dashed red and $R(Q)$ in solid blue. Their crossing gives the steady state value for Q. The parameterisation is the one used in the numerical work below. The right panel plots the same, and the thick green line plots $R(Q)$ for a value of Λ 20% higher than the baseline value, leading $R(Q)$ to shift to the left and steady state Q to be lower.

[1] We can verify that this equation reduces to the standard discounted sum from a normal matching model if we remove the financial friction by setting $\Lambda = 0$ so that the experts can't steal anything, and hence aren't constrained in equilibrium: If $\Lambda = 0$ then the expert is never constrained, so $\lambda = 0$. We can verify in this case that $\nu \equiv 1$, which means that $\tilde{\Omega} = \Omega$ (i.e. the expert's SDF is just the consumer's SDF). Finally this means that the denominator is equal to one, leaving: $Q(s) = \mathrm{E}\left[\Omega(s', s)\left(z' - w(s') + (1 - \rho_x)Q(s')\right)\middle|\, s\right]$

rough story where the model with financial frictions is either a dampened or amplified version of the standard matching model, depending on the cyclical behaviour of the financial friction. If the financial friction binds less in booms ($\lambda(s)$ countercyclical) then the model will be an amplified version of the standard matching model. This is because in a boom not only is the value of a match higher, but also now the experts are less constrained and can fund more matches. This can happen if asset prices are sufficiently procyclical so that expert net worth increases enough in booms to relax their borrowing constraints. On the other hand, if the financial friction binds more in booms ($\lambda(s)$ procyclical) then the model will be a dampened version of the standard matching model. This can happen if asset prices are not sufficiently procyclical, so that in a boom expert net worth does not increase enough. In this case experts will feel more constrained in a boom, because they want to invest to take advantage of higher productivity but do not have sufficient net worth. While both cases are possible, depending on how you calibrate the model, we will see in the numerical section that a model calibrated to match the volatility of asset prices in the data will deliver amplification.

Finally, the complete markets financial frictions model delivers a discounted sum which is similar to the standard matching model up to a constant wedge. Since this wedge is constant, we should not expect any drastic cyclical differences between the complete markets and standard model.

6 Analytical results

In this section I present analytical results for the steady states of the models, as well as for a special case of the fully dynamic model. These serve to illustrate the key mechanisms of the models in a sharp and transparent manner before I move on to the numerical results. In particular, I am able to prove that financial frictions must both increase unemployment in steady state, and increase its volatility. Using a sequence of proofs I show how it is crucially the transmission of net worth, and experts' inability to insure against it, which causes the divergence between the models with and without financial frictions.

6.1 Steady state results

In this section I compare how the steady states are determined in the models with and without financial frictions. I focus on the non-stochastic steady states, which means that the models with and without complete markets become identical. I also abstract from wage setting and focus on steady states conditional on a given, fixed wage. I denote steady state variables by omitting the explicit depending on the state, s. The determination of the steady state in the model without

financial frictions can be summarised in the following three equations:

$$Q = \frac{\beta(z-w)}{1-\beta(1-\rho_x)}$$

$$\downarrow$$

$$\theta = \left(\frac{\psi_0 Q}{\kappa z}\right)^{\frac{1}{\psi_1}} \tag{35}$$

$$\downarrow$$

$$K = \frac{\psi_0 \theta^{1-\psi_1}}{\psi_0 \theta^{1-\psi_1} + \rho_x} \tag{36}$$

As indicated by the arrows, we can solve the equations sequentially. The discounted match surplus is given on the top, which gives us the tightness required by free entry, which gives us the steady state level of employment.

Solving for the steady state in the model with financial friction is more complicated, but while it is hard to get analytical solutions we can characterise the equilibrium graphically. The definition of expert leverage gives us $QK = \phi N$, which we can interpret as the intersection of the supply and demand for matches. The right hand side gives us the total resources experts are putting towards buying old and new matches: net worth multiplied by leverage. The left hand side tells us that this must be spent on the total value of matches in the economy: their price multiplied by their quantity. To solve for equilibrium I note that we can express both the left and right hand sides solely as functions of Q. The left hand side, which I interpret as match supply, uses equations (35) and (36), which are common with the standard model:

$$L(Q) \equiv QK = Q\frac{\psi_0\left(\frac{\psi_0 Q}{\kappa z}\right)^{\frac{1-\psi_1}{\psi_1}}}{\psi_0\left(\frac{\psi_0 Q}{\kappa z}\right)^{\frac{1-\psi_1}{\psi_1}} + \rho_x}$$

Notice that $L'(Q) > 0$, so our supply curve is upwards sloping. The demand curve can be shown to be downwards sloping because both leverage (ϕ) and net worth (N) are decreasing in Q. The proof is left to the appendix, but the intuition is simple. The steady state return on investing is $r_k = (z-w)/Q + 1 - \rho_x$, which is decreasing in Q. A lower return reduces expert value and hence the maximum leverage allowed by the borrowing constraint, hence $\phi'(Q) < 0$. Lowever returns and leverage both reduce expert earnings and hence steady state net worth, so $N'(Q) < 0$. Hence $R(Q) \equiv \phi N$, with $R'(Q) < 0$.[2]

[2] One very important issue here is uniqueness of equilibrium. Gertler & Karadi (2011) do not discuss this, but it is actually possible to for their model to feature two steady states for some parameterisations, and thus admit the

As shown in Figure 1, equilibrium Q is at the intersection of the supply and demand curves. Once we have Q, equilibrium tightness and employment can be calculated as in the model without financial frictions. We can also use the graph to prove some results using comparative statics, and illustrate how the financial friction affects the economy in steady state. Firstly, it is worth asking how the steady state of the financial frictions economy compares to the steady state of the standard matching model when they are given the same parameter values:

Proposition 1. *In a steady state where the financial friction binds, employment is strictly lower than in the model without financial frictions.*

Proof. If the financial frictions model had employment weakly greater than the model without financial frictions then tightness would also be weakly greater, and by (8) so too would be the steady state match price, Q. But since $r_k = (\bar{z} - \bar{w})/Q + 1 - \rho_x$ and $r_k = r = 1/\beta$ in the model without frictions, this would imply $r_k \leq r$ in the financial frictions model, in which case the financial friction does not strictly bind. □

This result is perhaps to be expected. The intuition is quite simple. In the steady state of the standard economy, the return on capital is equal to the interest rate: $r_k = r$. If the financial frictions economy had the same level of employment as the standard economy it would have to be the case that $r_k = r$ in the financial frictions economy too. However, if $r_k = r$ then the financial friction does not strictly bind, because experts do not make positive profits on lending, and are hence indifferent about lending more. The next proposition establishes some comparative statics within the financial frictions models:

Proposition 2. *In a steady state where the financial friction binds, an increase in the amount that experts can expropriate, Λ, or a reduction of the equity injection given to new experts, w_e, reduces the steady state match price, Q, and hence employment, K.*

Proof. Both of these changes leave the supply curve, $L(Q)$, unchanged, while shifting the demand curve, $R(Q)$, to the left. For a given Q, reducing w_e shifts $R(Q)$ to the left by reducing N while leaving ϕ unchanged, while increasing Λ reduces ϕ and hence consequently also reduces N. □

Both of these changes reduce the funds that experts can allocate to purchasing matches. Increasing Λ allows experts to steal more, and hence requires them to have lower leverage, and reducing

possibility of multiple equilibria selected by sunspots. To see this, note that in steady state we solve for ϕ and ν from the expert equations (37) and (38) for a given value of r_k. Combining the two equations gives a quadratic equation in ϕ, giving two different solutions. Why is there multiplicity here? The intuition is simple. Leverage is limited by expert value, but expert value is higher when you're allowed more leverage. This multiplicity is a potentially interesting source of fluctuations, however for my baseline calibration there is actually only a unique steady state, around which I linearise.

w_e directly reduces expert net worth. Both of these shift the demand curve to the left, as shown in Figure 1, reducing steady state employment.

6.2 Dynamic results

In this section I analytically explore some features of the dynamic equilibria in a special case of the models with log utility and wages proportional to productivity. This specialisation is useful because it implies a particularly simple equilibrium in the model without financial frictions: unemployment is constant over the business cycle in response to productivity shocks. This stark result allows us to characterise what elements of the financial frictions model bring to the table, because under certain conditions we can replicate this result in the financial frictions economy. The following proposition establishes the initial result:

Proposition 3. *If wages are proportional to productivity and the household has log utility, then the model without financial frictions has an equilibrium where unemployment is constant in response to productivity shocks. The price of a match is proportional to current productivity:* $Q(s) = \bar{Q}z$.

This result is standard, and also relies on my assumption that vacancy posting costs are proportional to current productivity. Following a positive productivity shock the linear wage ensures that the value of a match rises proportionally to current productivity, as does the posting cost. Hence there is no incentive to change vacancy posting. The stock market does move though, and stock prices are proportional to current productivity.

Does adding financial frictions break this result? In the end it will, but I show that the simultaneous presence of several elements is required. Firstly a volatile stock market is not enough on its own. After all, the model without financial frictions generates movements in stock prices. The second element we need is an interaction between stock prices and expert net worth.

We can see this by considering versions of the financial frictions model which explicitly shut down the interaction between net worth and stock prices. The first version I consider is one where experts pay out all of their net worth as dividends each periods: $\sigma = 0$. This means that expert net worth each period is simply the net worth of new experts, $w_e z$, which is assumed proportional to productivity. In this case we can prove the following proposition:

Proposition 4. *If wages are proportional to productivity, the household has log utility, and experts pay out all of their net worth as dividends each period ($\sigma = 0$), then the model with financial frictions has an equilibrium where unemployment is constant in response to productivity shocks. The price of a match is proportional to current productivity:* $Q(s) = \bar{Q}z$.

This model features a moral hazard problem which restricts leverage, and incomplete markets since experts can only borrow risk free. However, we still recover the result that unemployment is

constant. Why is this? As in the model without financial frictions, the match price being proportional to productivity allows vacancy posting to be constant since posting costs are also proportional. However, we now need to understand why the match price being proportional is allowable even with the financial friction. Several elements come together to make this possible. Firstly, experts optimally choose constant leverage in this model. This is because the experts excess return on lending, once discounted with log utility and consumption which is (in equilibrium) proportional to z, becomes constant. With constant leverage, net worth being proportional to productivity means that even though positive productivity shocks make experts richer, they end up spending this on the increased match price and posting costs, which are also proportional. In other words given the increase in asset prices, experts do not have any left over cash to spend on increasing total matches.

The key to the result is really that net worth is proportional to z, and leverage is constant. Since net worth is only as volatile as the productivity shock there is no financial accelerator. The same result emerges from the model with complete markets, even if $\sigma > 0$, because the contingency of debt leads agents to optimally make net worth proportional to z:

Proposition 5. *If wages are proportional to productivity, then the model with financial frictions and state contingent debt has an equilibrium where unemployment is constant in response to productivity shocks. The price of a match is proportional to current productivity: $Q(s) = \bar{Q}z$.*

In this model experts receive a higher, leveraged return when there are good productivity shocks, leading to the possibility that net worth is more volatile than productivity. However, the contracts they choose offset this in equilibrium, since they choose to structure their contingent claims to repay more in good states than bad, leading to net worth again only being as volatile as productivity.

The final result considers the case of incomplete markets with $\sigma > 0$ and shows that, in contrast to the cases above, it is not possible to generate an equilibrium with constant unemployment:

Proposition 6. *The model with financial frictions with $\sigma > 0$ does not have an equilibrium where unemployment is constant in response to productivity shocks when wages are proportional to productivity and utility is log.*

Proof. The proof of this proposition is a simple disproof: we conjecture that the model does have an equilibrium with constant unemployment and show this violates one of the equilibrium conditions. Constant unemployment requires constant market tightness, which requires, via (8), that Q is proportional to z: $Q(s) = \bar{Q}z$. As in the other models, this means that experts optimally choose constant leverage, $\bar{\phi}$. Denote by $\bar{K}$ the constant level of employment, and (15) requires that $N = \bar{Q}\bar{K}z/\bar{\phi}$. In other words, for experts to have the right amount of net worth, on aggregate, to purchase the stock of matches requires that net worth be proportional to productivity. However,

we can easily show that this leads to a contradiction since debt is not state contingent. Plugging in $Q(s) = \bar{Q}z$ to the equation governing total expert net worth, (14), gives:

$$N = \sigma \left(\left(z - \bar{w}z + (1 - \rho_x)\bar{Q}z \right) \bar{K} - D \right) + (1 - \sigma) \, w_e z$$

This is not proportional to z due to the fixed stock of debt, D, which does not vary with z. Hence it cannot be the case that $N(s) = \bar{N}z$. □

Since the model without financial frictions and the model with complete markets have zero volatility of unemployment over the cycle, and we have proven that the incomplete markets model must have positive volatility, I have proven that the incomplete markets model is more volatile in this special case. I have also shown the crucial role of net worth in this mechanism. Of course, one should be sceptical of analytical results derived from special cases, and this is certainly true here. In general we know that financial frictions can deliver either amplification of dampening, as I discussed in the previous section. To this end, I present calibrated numerical results in the next section.

7 Numerical results

In this section I present perturbation numerical result to analyse the quantitative significance of the ideas presented in the precious sections. In particular, I calibrate the model to assess whether it is able to match key features of the data. To test the model, I will compare its ability to generate volatility in market tightness and unemployment to the data, once the model is calibrated to match other moments of the data. I will be interested in comparing the ability of the models with and without financial fractions to generate volatility in unemployment. Thus the calibration of the financial frictions parameters will be important as they will determine how powerful financial frictions can be in a quantitative sense. For this, I take an approach similar in spirit to Winkler (2015). He chooses certain parameters of his model in order to match properties of asset prices in the data, and I do the same here. In particular, I will choose the parameters governing financial frictions to match certain asset price moments.

Another key issue in assessing the quantitative performance of my model is the current controversy over how to calibrate wages in the search and matching model. As discussed further below, the average level of wages is important in determining the volatility of unemployment. This is true in the baseline search and matching model, and is also true in my extension with financial frictions. I thus perform robustness checks for different values of this parameter, as well as various financial frictions parameters. I solve the model using first order log-linearisation in Dynare. The model is solved and simulated at a monthly frequency, and I take simple averages to compute quarterly statistics.

7.1 Data moments

In this section I describe the data I aim to test my model against. Table 1 presents the covariances and autocorrelations for seven key US time series. The data is quarterly, and covers the period 1951Q4 to 2014Q2. All data are seasonally adjusted, logged, and HP-filtered with smoothing parameter 10^5. Any data which is collected with monthly frequency are converted to quarterly figures by a simple average.

7.1.1 Non-financial moments

My measure of unemployment, u, is the Civilian Unemployment Rate in percent from the Current Population survey. Vacancies, v, is the composite Help Wanted Index of Barnichon (2010), available from the author's website.[3] Market tightness, θ, is calculated as the ratio of the Help Wanted Index and Total Unemployment (thousands) from the Current Population Survey. Real wages, w, are calculated as total labour compensation per employee from the national accounts. To measure this I first construct the labour share (as detailed in my second chapter) and then measure wages per employee as the labour share multiplied by output over employment. Output, y, is chained real GDP taken from Line 1 of Table 1.1.6 of the National Income and Product Accounts from the Bureau of Economic Analysis. Labour productivity, z, is my measure of output divided by total employment. Total employment is measured as Total Nonfarm Employees from the Current Employment Statistics survey.

7.1.2 Constructing a measure of asset prices

Finally I need to construct a measure of asset prices. Ideally, this should be as close as possible to the definition of the asset price in my model, Q, which is the price of the entire equity stake in a firm with a single worker. One issue that arises is in the treatment of firm assets in the data, which the model abstracts from. I will discuss this in more detail below. I use two measures of equity prices, and I opt to measure in both the data and my model the quantity Q/z, which is thus the price of equity in a single worker firm scaled by labour productivity. This can be conveniently measured in the data as the ratio of the total real value of equity in the economy to real GDP.[4] I use two different measures of the total nominal value of equity. The first is the closing price of the S&P 500 index, collected from Yahoo finance.[5] The second is a measure of total market capitalisation of the

[3] At the time of writing, the data is available at `https://sites.google.com/site/regisbarnichon/research`

[4] To see this, note that $TE/y = (TE/n)/(y/n) = \tilde{Q}/z$ where TE is a measure of total equity value, n is employment, and $\tilde{Q}$ is the average equity value per worker in the data.

[5] Series S&P500 (^GSPC). At the time of writing this is available at `https://finance.yahoo.com/q/hp?s=%5EGSPC+Historical+Prices`

US economy from the Flow of Funds accounts. I use Nonfinancial Corporate Business; Corporate Equities; Liability.[6] Both nominal values of equity are deflated using the GDP deflator, taken from Line 1 of Table 1.1.4 of the NIPA accounts. I report the results only for my second measure of equity, but the moments of the two series are remarkably similar: the log standard deviations of the HP-filtered market capitalisation and S&P 500 series are 0.1516 and 0.1511 respectively. Their similarity is heartening, especially since one measure contains financial firms and one does not, and one might think that it would be appropriate to strip out financial firms from my measure of equity given that I split out experts from the rest of the economy in my model. Stripping out the value of firm assets from the data is more challenging, and I do not undertake this task here. Instead I choose to use the raw measures of asset prices as my primary data, and discuss the effects and challenges of attempting to split out the value of firm assets from the data in my robustness section.

Table 1: Data moments

	u	v	θ	w	y	z	Q/z
Standard deviation	0.195	0.188	0.371	0.016	0.025	0.015	0.152
Autocorrelation	0.947	0.941	0.948	0.930	0.939	0.907	0.845
Correlation	1	-0.889	-0.973	-0.237	-0.864	-0.193	-0.293
	–	1	0.970	0.097	0.803	0.202	0.270
	–	–	1	0.177	0.856	0.211	0.297
	–	–	–	1	0.526	0.687	0.088
	–	–	–	–	1	0.511	0.217
	–	–	–	–	–	1	0.142
	–	–	–	–	–	–	1

All data are quarterly, logged and then HP-filtered with smoothing parameter 10^5.

7.2 Baseline calibration

In order to test my model I first calibrate the parameters. Some parameters are calibrated to steady state targets, while others are calibrated to match certain moments of the data. After solving the model I compute moments which are comparable to the moments calculated in the data. Specifically, I simulate the model for a length of time equal to the length of my data one hundred times, and calculate the means and standard deviations of each moment across the repetitions. A summary table of calibration tables can be found in the appendix.

[6] Series id: FL103164103.Q. Note that when using this measure, my measure Q/z is actually the Market Capitalisation to GDP ratio popularised by Warren Buffett.

Starting with the household, I choose a standard CRRA utility function, $u(c) = c^{1-\sigma_c}/(1-\sigma_c)$, and specialise to log utility. I choose the discount factor $\beta = 0.9966$ to match an annual risk free rate of 4.17%. The parameters of the productivity process are chosen so that, once log HP-filtered, the means of the standard deviation and autocorrelation of the log HP-filtered series match the data I presented in the previous section. I choose $\sigma_e = 0.0043$ to match the standard deviation of 0.0146 in the data, and $\rho_z = 0.98975$ to match the autocorrelation of 0.9068. I normalise steady state productivity, $\bar{z}$ to one.

The labour market is parameterised following the calibration of Den Haan & Kaltenbrunner (2009), who report data giving a monthly job finding probability of $\lambda_w = 45.4\%$, vacancy filling probability of $\lambda_f = 33.8\%$ and unemployment rate of $u_s s = 5.7\%$. This allows me to pin down steady state tightness as $\theta_s s = \lambda_w/\lambda_f$. I assume a standard value of $\psi_1 = 0.5$ (Petrongolo & Pissarides, 2001) for the matching function elasticity. This allows me to pin down match efficiency as $\psi_0 = \lambda_f \theta_{ss}^{\psi_1} = 0.3917$. The job separation rate is picked to equate the flows of workers in and out of unemployment in steady state, giving $\rho_x = \lambda_w u_{ss}/(1-u_{ss}) = 0.0274$.

Real wage flexibility is set to $\gamma = 0.7$ following the empirical discussion in Michaillat (2012). This corresponds to an elasticity of wages to productivity of 0.7, consistent with the empirical evidence from job movers of Haefke, Sonntag & Van Rens (2007). I also set the steady state real wage, $\bar{w}$ following Michaillat (2012). Based on empirical estimates, he requires that the steady state recruiting cost, κ, is equal to 0.32 of a workers steady state wage. This allows me to jointly solve for $\bar{w}$ and κ from equations (8) and (10), given a value of the steady state return on matches, $r_{k,ss}$, which I detail below. This gives values $\bar{w} = 0.9709$ and $\kappa = 0.3107$.

The expert parameters are calibrated to match asset price moments. There are three parameters to choose: the fraction of experts who survive each period, σ, the fraction of assets the experts can steal, Λ, and the equity injections given to new experts, w_e. These are jointly chosen to match three asset pricing moments: the equity premium and the standard deviation and autocorrelation of asset prices. I target the values for the standard deviation and autocorrelation of asset prices from the data in Table 1. For the equity premium I instead target a value lower than that found in the data, targeting a 1% premium of yearly equity returns over the risk free rate in steady state. This value corresponds to the premium in Gertler & Karadi (2011). The presence of financial frictions means that my model generates an equity premium even in the non-stochastic steady state. The equity premium in the data presumably reflects this wedge, as well as compensation for risk. Since there is no risk in my non stochastic steady state I do not want to attribute this part of the data to this moment, and hence choose a lower value. I use a numerical minimisation routine to find the values of the parameters which achieve these values of the moments, leading me to choose $\Lambda = 0.4854$,

$\sigma = 0.9770$, and $w_e = 0.3026$. These values correspond to experts surviving 3.62 years on average, and having steady state leverage of 2.23. The expert sector pays out a fraction $1 - \sigma = 0.0230$ of its net worth as equity per month.

The model without financial frictions is calibrated using the same procedure as above, but without the financial frictions components. This means that there is a slight difference difference in the calibrated values of $\bar{w}$ and κ between the two models. The procedure to choose the values of these two parameters is exactly the same as for the financial frictions model, imposing $r^k_{ss} = 1/\beta$. For the complete markets model I take the values of all of the parameters as the calibrated values from the incomplete markets model. Note that I am thus not calibrating the three models to the same targets: the incomplete markets model is calibrated to match asset price moments, whereas the other two models are not calibrated to match these moments. In this sense I am not providing a test across the three models. I am only testing the ability of the incomplete markets model to match the volatility of unemployment once it is properly calibrated. The other two models are not tested, and their solutions are only provided to serve as references against which to compare the incomplete markets model.

7.3 Model evaluation

7.3.1 Moments

Table 2 reports moments calculated from simulating the incomplete markets model which are comparable to the empirical moments presented in Table 1. I simulate the model 100 times for a length of time equal to the length of the data sample, and calculate the same moments I calculated in the data for each of these samples. I then report the means and standard deviations (in parentheses) of these moments. The volatility and autocorrelation of productivity and asset prices is the same as the data by construction since these were calibrated to fit the data.

Given that the model was not calibrated to match the volatility of unemployment, the performance is surprisingly good. The model generates an average standard deviation of labour market tightness of 0.307, which is 83% of that observed in the data. Similarly for unemployment, the model is able to generate 72% of the volatility observed in the data. In the current calibration this represents a significant improvement over the model without financial frictions, which only delivers 61% of the volatility of tightness from the data. Moments for the model without financial frictions are available in Table 4 in the appendix. My calibration features a relatively high (on average 97% of productivity) and sticky wage which explains why the model without financial frictions performs relatively well compared to Shimer's (2005) calibration. However, as I discuss in my robustness, given my calibration strategy this does not actually impact on the ability of my model with financial

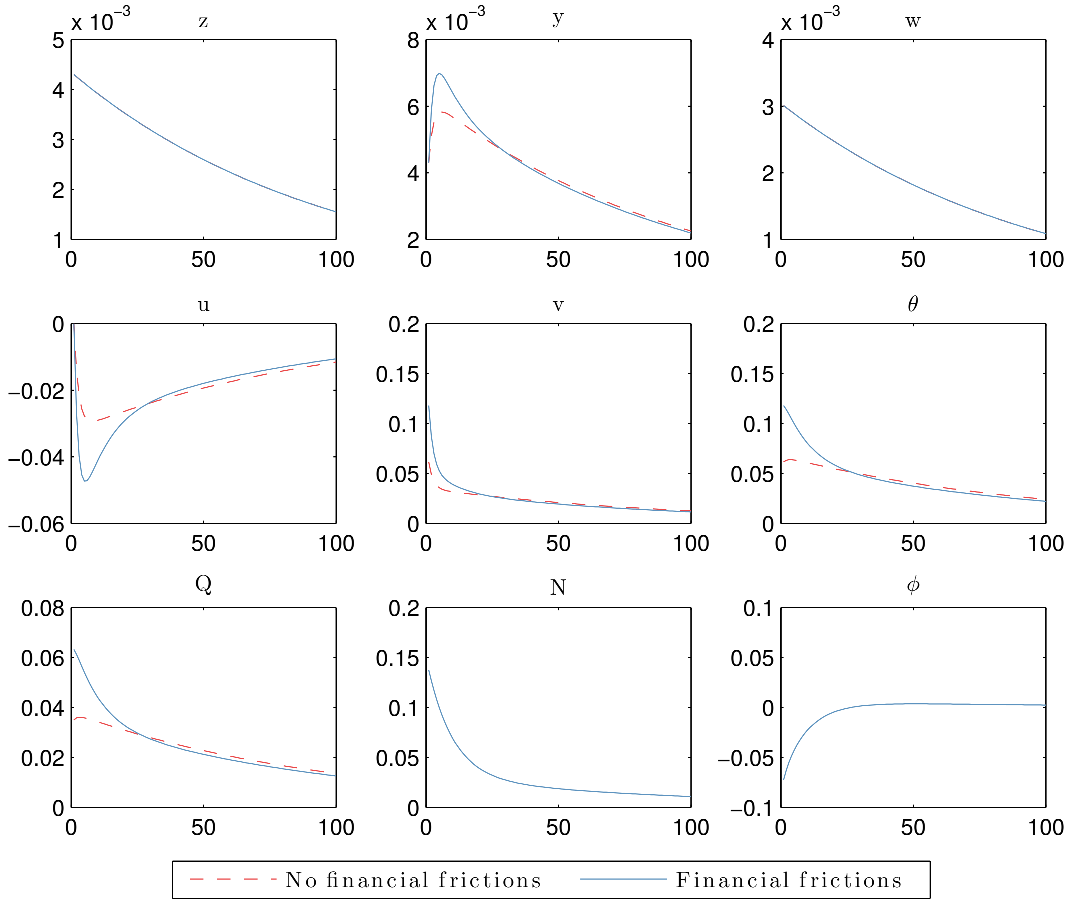
Table 2: Simulated moments of the incomplete markets financial frictions model

	u	v	θ	w	y	z	Q/z
Standard deviation	0.138	0.180	0.303	0.010	0.023	0.015	0.152
	(0.017)	(0.017)	(0.036)	(0.002)	(0.003)	(0.002)	(0.018)
Autocorrelation	0.890	0.709	0.845	0.907	0.914	0.907	0.845
	(0.025)	(0.056)	(0.034)	(0.026)	(0.023)	(0.026)	(0.034)
Correlation	1	-0.812	-0.938	-0.952	-0.980	-0.952	-0.938
		(0.001)	(0.001)	(0.001)	(1×10^{-4})	(0.001)	(0.001)
	–	1	0.964	0.879	0.864	0.879	0.964
			(3×10^{-4})	(0.001)	(0.001)	(0.001)	(3×10^{-4})
	–	–	1	0.956	0.960	0.956	1.000
				(0.001)	(0.001)	(0.001)	(0)
	–	–	–	1	0.994	1.000	0.956
					(2×10^{-4})	(0)	(0.001)
	–	–	–	–	1	0.994	0.960
						(2×10^{-4})	(0.001)
	–	–	–	–	–	1	0.956
							(0.001)
	–	–	–	–	–	–	1

I simulate the model 100 times for 526 months, corresponding to the length of the data sample. I convert the data to quarterly frequency as per Michaillat (2012). All data are quarterly, logged and then HP-filtered with smoothing parameter 10^5. *The numbers in parentheses are the standard errors of the sample statistics.*

frictions to match the data significantly. The moments and IRFs for the complete markets model are not reported since they are very similar to the model without financial frictions. For example, the average standard deviation of tightness is 0.2255 in the model without financial frictions, and 0.2212 in the complete markets model. Where the model does not perform well is the correlation of stock prices with real variables. The correlations in the model are all very high, whereas in the data they tend to be much lower. This likely reflects the absence of other shocks in the model which could introduce independent volatility into both variables.

Figure 2: Impulse responses to a one standard deviation innovation to productivity



Impulse responses are log deviations from steady state. One period corresponds to one month.

7.3.2 Impulse response functions

Figure 2 plots the impulse response functions of both the financial frictions model and the standard model for comparison. That financial frictions give amplification is clear from the tightness and unemployment panels: the peak response of tightness is almost twice as high in the financial frictions model. The impulse responses reveal the mechanisms behind the model's financial accelerator, which can be traced out as follows. Recall that experts are financial constrained, and would like to fund extra vacancies on the margin but cannot afford to. A positive productivity shock allows them to do so, providing an initial increase in vacancies, and hence an increase in market tightness. An increase in market tightness makes it harder for firms to hire workers, which increases the value of being an existing firm, hence pushing up the stock price of existing matches. This is exactly the arbitrage argument implied by equation (8). Since existing matches are owned by the expert sector this pushes up experts' net worth, which allows them to fund even more vacancies, and the cycle continues.

7.4 Robustness

7.4.1 Robustness to alternative parameterisations

In this section I perform robustness checks on several parameters. This serves firstly as a check on my results, but also highlights the key role of asset price volatility in my model. Recall that arbitrage between old matches and vacancies implied the following relationship between the volatilities of market tightness and asset prices:

$$\sigma\left(\log\theta\right) = \frac{1}{\psi_1}\sigma\left(\log\frac{Q}{z}\right)$$

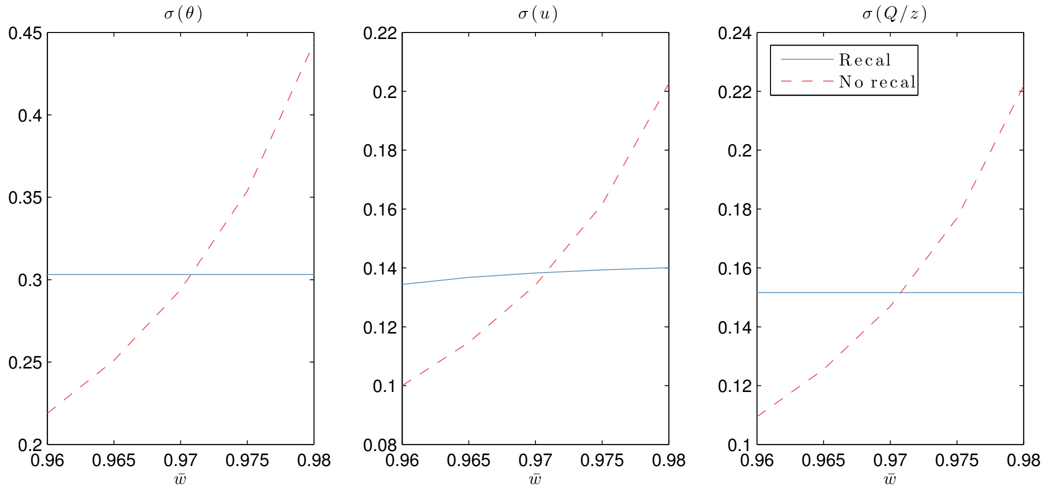
This implies that any model in which this arbitrage equation holds will generate the same standard deviation of tightness as long they share the same volatility of stock prices, and hence that any model calibrated to match the data on the volatility of asset prices will generate 82% of the volatility in tightness in the data, as does my baseline model. This has strong implications for my robustness checks, since it implies that if even if I change a parameter, as long as I recalibrate the financial sector to match the volatility of asset prices the model will still perform just as well at explaining the volatility in tightness.

I illustrate this by varying three parameters. I first explore the effects of varying the average level and stickiness of the real wage. These are key parameters for which there is still debate on how to calibrate. I also explore different values of the steady state equity premium. I do this because my calibration strategy involved only targeting an arbitrary fraction of the equity premium from the data, and I show below that the results are robust to alternative values. After perturbing these parameters, I report the solution to two variants of the model. The first variant holds all other parameters at their original values. In this solution, perturbing one parameter will thus affect the volatility of asset prices and tightness. In the second variant, I perturb the parameter but also adjust the financial sector parameters in order to maintain the volatility of asset prices at its original level.[7]

Figure 3 plots the results of this exercise for the average wage, $\bar{w}$. The dashed red line shows that, without recalibrating the financial parameters, a higher average wage pushes up the volatility of tightness, unemployment, and asset prices as we would expect. However, once we recalibrate to keep the volatility of asset prices constant, this effect disappears. The volatility of both asset prices and tightness remain constant at their original level, the former by construction and the latter due to the strong arbitrage arguments made above. The effect on unemployment volatility is virtually, but not entirely removed since unemployment is a stock and its volatility depends also on the volatility of its lagged values.

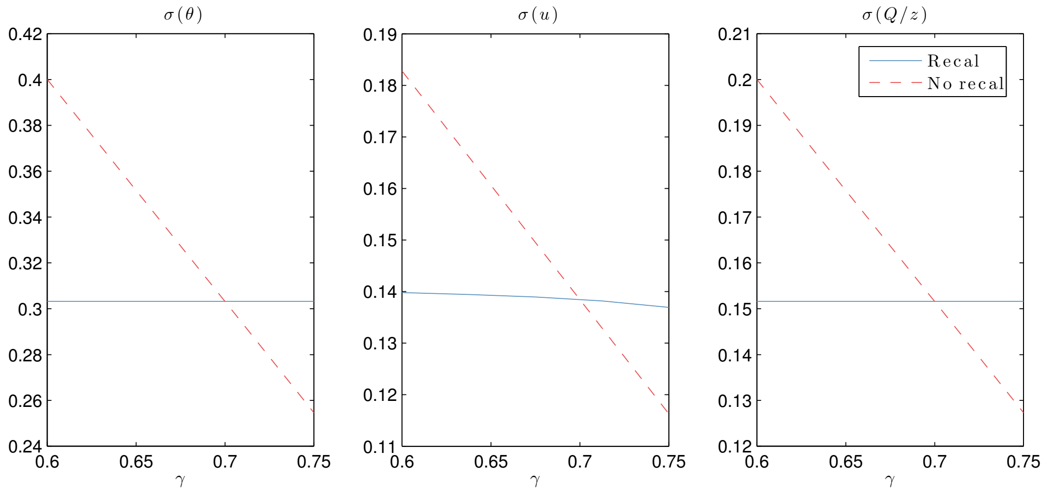
[7] Specifically, I keep σ at its original level, and adjust Λ and w_e in order to maintain the volatility of asset prices and the steady state equity premium at their original levels. In principle I could also adjust σ so that I also maintain the autocorrelation of asset prices. This exercise is slightly more computationally demanding, and the results are similar.

Figure 3: Robustness: $\bar{w}$



Standard deviations are computed as in the original model: these are the average standard deviations across 100 replications for different parameter values. Variables are logged and HP-filtered. The solid blue line gives volatilities when the financial variables are recalibrated, the dashed red line when they are not.

Figure 4: Robustness: γ

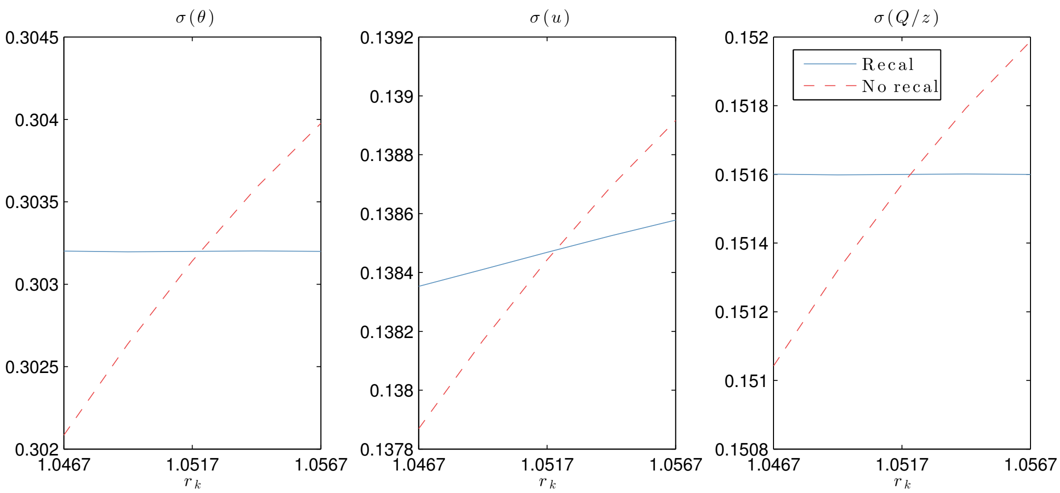


Standard deviations are computed as in the original model: these are the average standard deviations across 100 replications for different parameter values. Variables are logged and HP-filtered. The solid blue line gives volatilities when the financial variables are recalibrated, the dashed red line when they are not.

The story is the same for the changes in wage stickiness, γ, which are plotted in Figure 4. Increasing γ reduces wage stickiness and hence reduces the volatility of labour market variables.

Interestingly, we can view this from a financial angle as well. Reducing wage stickiness reduces the responsiveness of profit to shocks, which reduces the sensitivity of net worth to shocks and hence dampens the financial accelerator. This exercise reveals an interesting interaction between wage stickiness and the financial accelerator: wage stickiness makes the financial accelerator more severe. We can see this by comparing the difference between the volatilities of tightness in the models with and without financial frictions for different values of γ. With the baseline value of $\gamma = 0.7$ this difference is 0.078, but if stickiness is increased by setting $\gamma = 0.6$ the difference widens to 0.107, showing that increases in wage stickiness generate extra volatility in the financial frictions model above the model without financial frictions.[8] The same argument holds for increases in the average wage: these increase the gap between the volatility of the models with and without financial frictions. As before, once we recalibrate the financial parameters, the effects disappears for θ and Q/z, and is severely diminished for u.

Figure 5: Robustness: steady state r_k



Standard deviations are computed as in the original model: these are the average standard deviations across 100 replications for different parameter values. Variables are logged and HP-filtered. The solid blue line gives volatilities when the financial variables are recalibrated, the dashed red line when they are not. The horizontal axis gives the steady state yearly value of r_k.

Finally, I perform robustness for my assumed value of the steady state equity premium. In this exercise the recalibrated model is calibrated to give the original volatility of asset prices while

[8]This argument is similar to the argument of Schoefer (2015) within a financial accelerator context. He makes the argument within firms: sticky wages make firm cash-flow more volatile leading to volatility in the cash available for hiring. I make the argument that sticky wages make asset prices more volatile, further impacting the volatility of the net worth of experts.

matching the new equity premium. This exercise reveals that my choice of equity premium target is not particularly important, and that varying the target within a one percentage point range has very small effects on the volatilities of asset prices and tightness even if I don't recalibrate the other financial parameters.

7.4.2 Robustness of asset price data

Checking the robustness of my measure of asset prices is an important exercise, since, as I pointed out above, the ability of my calibrated model to match the volatility of tightness depends crucially on the volatility of asset prices. I have already discussed the robustness of my data to the inclusion or exclusion of financial firms, and in this section I attempt to address another concern: the effect of controlling for the value of firms' assets.

Stripping out the value of firms' assets from the data is challenging. To see why it is important, remember that my model abstracts from capital, and firms' only assets are their relationships with employees. The productivity process is calibrated to therefore implicitly include capital movements, and firms don't own any capital. In the data the total equity value of firms would contain both the value of their relationships with workers and asset ownership, as well as any other sources of value such as tax shields or intangibles. Ultimately splitting out these various sources reliably is a challenging feat, and data limitations place bounds on our ability to do this. For example, Hall (2014) points out that attempts to do so lead to large periods when the stock market value of firms falls far below the measured value of firms' plants and equipment. These concerns aside, I make an attempt here to investigate how robust my results are to doing so. Since we would expect the value of firms' assets to be procyclical, due to both price and quantity effects, controlling for this could reduce the volatility of asset prices compared to the baseline measure.

Measuring the individual components of firm value is hard, so I first present an example showing how mismeasurement does not necessarily reduce my measure of the volatility of the worker-relationships component of firm value. One might expect this to be the case, especially if both relationship value and firms' asset values are positively correlated over the cycle: an increase in asset values should reduce the amount of an increase in total firm value we ascribe to worker relationships, and hence reduce its true volatility. However, since I am working with log volatilities this is not the case. Suppose we can cleanly split total firm equity value, E, into a component deriving from worker relationships, W, and a component deriving from the value of owned assets, K, giving $E = W + K$. I should calibrate my model to movements in W, but only measure E. To see how this mismeasurement need not reduce the true volatility of W, consider the special case where W and K are perfectly correlated such that we can write them both as loaded onto a common factor:

$W = wX$ and $K = kX$. In this case we have that $\sigma(\log W) = \sigma(\log K) = \sigma(\log E) = \sigma(\log X)$. Since we would expect that both W and K should be procyclical over the business cycle, to the extent that they will be highly correlated the above example suggests that the log-volatility of total firm value should be a good proxy for the log-volatility of the value of worker relationships.

In the case where the different components of firm value are not perfectly correlated the above does not exactly hold and we need to try and measure the components individually. I attempt to strip out the value of firm assets from my measure of firm equity by subtracting the net worth of non-financial firms from the market value of their equity. Their net worth is a measure of the value of their assets net of their liabilities, and is thus a measure of the replacement cost of a firm. My measure of firm net worth, NW, is Nonfinancial Corporate Business; Net Worth, Level from the Flow of Funds.[9] If I simply subtract this from the market value of firm equity (which is the measure I use in the calibration, and is constructed for the same sample as net worth) then I run in to the same problem as indicated by Hall (2014): for much of the sample, this leaves negative value. Specifically for over 85% of the quarters in my sample. Regardless of whether this is correct or reflecting of measurement issues, this leaves me with the immediate problem that I cannot take logarithms of the adjusted series to compare its logged, HP-filtered volatility to my original series. Given this issue, my first check is to simply compare HP-filtered volatilities without taking logs. To do this I compute $\sigma(E/Y)$ and $\sigma(\tilde{E}/Y)$ where E refers to the market value of firm equity in the data, and $\tilde{E} = E - NW$ is the equity value series minus the net worth series. Recall that measuring this ratio gives a series comparable to Q/z in my model. If I do this, I find $\sigma(E/Y) = 0.1191$ and $\sigma(\tilde{E}/Y) = 0.1318$. In other words, rather than reducing the volatility of measured asset prices, as we would expect, the correction actually increases it slightly. This is surprising since we would expect net worth to be procyclical. From the 90s onwards this is certainly the case, however earlier in the sample the series (once HP-filtered) displays a slight countercyclicality, which could be due to measurement issues.

The second correction I do aims to take seriously the issue of the many negative values of firm value once the value of assets is stripped out. In particular, it is plausible that the way assets are market to market and valued makes the series not easily comparable. To try and adjust for this, I construct the series $\tilde{E}_\alpha = E - \alpha NW$ for different values of $\alpha \in (0, 1]$, the idea being that the scaling of the two series might not be comparable. $\alpha = 0$ corresponds to my original series, and $\alpha = 1$ corresponds to the corrected series above. This generates a minimum value of $\sigma(\tilde{E}_\alpha/Y) = 0.1191$ and a maximum of $\sigma(\tilde{E}_\alpha/Y) = 0.1318$. Additionally α can be chosen to match average values of the series. For example, $\alpha = 0.485$ generates a series for $\tilde{E}_\alpha$ which implies that on average

[9] Series id: FL102090005.Q

20% of total equity value derives from sources other than firms' assets, and generates a value of $\sigma(\tilde{E}_\alpha/Y) = 0.1217$. Overall, this exercise, while imperfect, does not immediately suggest that any large overstatement of the correct asset price for my model is induced by ignoring firms' assets.

Other sources are less easy to account for. Intangible capital and the value of tax shields are both hard to measure, and would require model based frameworks in order to estimate their contribution to firm equity value. Ultimately, the importance of addressing these concerns cannot be overstated, but I leave the exercise to future work.

8 Summary

In summary, I introduce financial frictions into the labour market matching model, and study interactions between the two frictions. I demonstrate a feedback between asset and labour markets which amplifies the model's response to exogenous shocks. Shocks which increase expert net worth allow experts to fund more vacancies, raising market tightness and lowering the ease with which firms can hire workers. This increases the value of being an existing firm, causing stock prices to appreciate. Since experts own firm stocks, this increases expert net worth further, amplifying the initial shock in a classic financial accelerator mechanism. I show how sticky wages, by making the stock market more volatile, amplify this financial accelerator, and how incomplete markets are required to generate the necessary volatility in expert net worth.

I derive an arbitrage equation in my model between equity prices and market tightness similar to the standard free entry condition. I show that as long as a matching model which shares this arbitrage condition is calibrated to match the volatility in asset prices in the data, it will always be able to generate 82% of the volatility in market tightness, and hence do a reasonable job at describing the volatility of the labour market. This is true in the standard matching model, and any variants where at least one agent is free to perform arbitrage between vacancies and existing matches. This holds regardless of the underlying source of shocks or the fractions of the volatility caused by sticky wages or financial frictions. Does this mean that I am simply assuming the result by calibrating my model to match the volatility of asset prices? In a sense I am, although it is worth remembering that there is is no ex ante guarantee that calibrating to asset prices will make the search and matching model work well. Indeed, it is actually very good news for the matching model that one of its key equations, the free entry condition, holds up so well against the data.

However, the key limitation of this approach is that while I have shown that a model with financial frictions can do a good job at explaining the data, I have not presented any direct evidence that financial frictions are the *only* mechanism which can do so. One could imagine that augmenting the model instead with other mechanisms to introduce volatility into asset demand and hence asset

prices would achieve the same end. Learning, habit formation, and non-time separable preferences have all been shown to improve asset pricing behaviour, and could all potentially serve as alternative explanations to financial frictions. Ultimately more work is needed to help disentangle which of these forces is responsible for the volatility in asset prices. However, if my paper has shown anything it is that once the correct source has been identified, the matching model has the potential to utilise it to generate a meaningful fraction of the unemployment volatility in the data.

References

[1] Adjemian, Stephane, Houtan Bastani, Michel Juillard, Frederic Karame, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto & Sebastien Villemot (2011). *Dynare: Reference Manual, Version 4.* Dynare Working Papers, 1, CEPREMAP.

[2] Barnichon, Regis, 2010. *Building a composite Help-Wanted Index.* Economics Letters, Elsevier, vol. 109(3), pages 175-178, December.

[3] Bernanke, Ben S, Gertler, Mark & Gilchrist, Simon, 1999. *The financial accelerator in a quantitative business cycle framework.* Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, pages 1341-1393 Elsevier.

[4] Caggese, Andrea & Vicente Cunat, 2008. *Financing Constraints and Fixed-term Employment Contracts.* Economic Journal, Royal Economic Society, vol. 118(533), pages 2013-2046, November.

[5] Carlstrom, Charles T, Fuerst, Timothy S, Ortiz, Alberto, Paustian, Matthias, 2014. *Estimating Contract Indexation in a Financial Accelerator Model.* Journal of Economic Dynamics and Control, Volume 46, September 2014, Pages 130-149.

[6] Christiano, Lawrence J, Trabandt, Mathias & Walentin, Karl, 2011. *Introducing financial frictions and unemployment into a small open economy model.* Journal of Economic Dynamics and Control, Elsevier, vol. 35(12), pages 1999-2041.

[7] Den Haan, Wouter J. & Kaltenbrunner, Georg, 2009. *Anticipated growth and business cycles in matching models.* Journal of Monetary Economics, Elsevier, vol. 56(3), pages 309-327, April.

[8] Dmitriev, Mikhail & Hoddenbagh, Jonathan, 2013. *The Financial Accelerator & the Optimal Lending Contract.* Mimeo.

[9] Farmer, Roger, 2012a. *The Stock Market Crash of 2008 Caused the Great Recession.* 2012 Meeting Papers 145, Society for Economic Dynamics.

[10] Farmer, Roger, 2012b. *Confidence, Crashes and Animal Spirits.* Economic Journal, Royal Economic Society, vol. 122(559), pages 155-172, 03.

[11] Favilukis, Jack & Xiaoji Lin, 2015. *Wage Rigidity: A Quantitative Solution to Several Asset Pricing Puzzles.* Mimeo, available at `https://sites.google.com/site/jackfavilukis/research`. Last accessed 27/03/2015.

[12] Gertler, Mark & Karadi, Peter, 2011. *A model of unconventional monetary policy.* Journal of Monetary Economics, Elsevier, vol. 58(1), pages 17-34, January.

[13] Haefke, Christian, Marcus Sonntag, and Thijs van Rens. 2007. *Wage Rigidity and Job Creation.* Institute for the Study of Labor (IZA) Discussion Paper 3714.

[14] Hagedorn, Marcus & Iourii Manovskii, 2008. *The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited.* American Economic Review, American Economic Association, vol. 98(4), pages 1692-1706, September.

[15] Hall, Robert, 2014. *High Discounts & High Unemployment.* Mimeo.

[16] Kiyotaki, Nobuhiro & Moore, John, 1997. *Credit Cycles.* Journal of Political Economy, University of Chicago Press, vol. 105(2), pages 211-48, April.

[17] Michaillat, Pascal, 2012. *Do Matching Frictions Explain Unemployment? Not in Bad Times.* American Economic Review, American Economic Association, vol. 102(4), pages 1721-50, June.

[18] Mumtaz, Haroon & Zanetti, Francesco , 2013. *The Effect of Labor and Financial Frictions on Aggregate Fluctuations.* Economics Series Working Papers 690, University of Oxford, Department of Economics.

[19] Petrongolo, Barbara & Christopher A. Pissarides, 2001. *Looking into the Black Box: A Survey of the Matching Function.* Journal of Economic Literature, American Economic Association, vol. 39(2), pages 390-431, June.

[20] Petrosky-Nadeau, Nicolas, 2014. *Credit, Vacancies and Unemployment Fluctuations.* Review of Economic Dynamics, Elsevier for the Society for Economic Dynamics, vol. 17(2), pages 191-205, April.

[21] Petrosky-Nadeau, Nicolas, Kuehn, Lars-Alexander & Zhang, Lu. *Endogenous Disasters and Asset Prices.* Charles A. Dice Center Working Paper No. 2012-1.

[22] Pissarides, Christopher A., 2000. *Equilibrium Unemployment Theory, 2nd Edition.* MIT Press Books, The MIT Press, edition 1, volume 1, number 0262161877, June.

[23] Quadrini, Vincenso, and Sun, Qi, 2015. *Credit and Hiring.* Mimeo, available at http://www-bcf.usc.edu/~quadrini/papers/UPpap.pdf. Last accessed 30/03/2015.

[24] Schoefer, Benjamin, 2015. *The Financial Channel of Wage Rigidity.* Mimeo, available at http://scholar.harvard.edu/files/schoefer/files/jmp_bschoefer.pdf. Last accessed 30/03/2015.

[25] Shimer, Robert, 2005. *The Cyclical Behavior of Equilibrium Unemployment and Vacancies.* American Economic Review, American Economic Association, vol. 95(1), pages 25-49, March.

[26] Winkler, Fabian, 2015. *The Role of Learning for Asset Prices, Business Cycles and Monetary Policy.* Mimeo, available at http://personal.lse.ac.uk/winklerh/jmp.pdf. Last accessed 27/03/2015.

A Equations and unknowns, incomplete markets model

A.1 Experts $[\phi, \nu, r_k, \lambda, K, D, N]$

Individual problem:

$$\nu(s) = \Lambda\phi(s)$$

$$\nu(s) = \mathrm{E}\left[\Omega(s', s)\left(1 - \sigma + \sigma\nu(s')\right)\left(\left(r_k(s', s) - r(s)\right)\phi(s) + r(s)\right)\big|\, s\right]$$

$$r_k(s', s) \equiv \frac{z' - w(s') + (1 - \rho_x)Q(s')}{Q(s)}$$

$$Q(s) = \frac{\mathrm{E}\left[\Omega(s', s)\left(1 - \sigma + \sigma\nu(s')\right)\left(z' - w(s') + (1 - \rho_x)Q(s')\right)\middle|\, s\right]}{\mathrm{E}\left[\Omega(s', s)\left(1 - \sigma + \sigma\nu(s')\right)\middle|\, s\right] r(s) + \frac{\lambda(s)\Lambda}{1+\lambda(s)}}$$

Aggregation:

$$Q(s)K'(s) = \phi(s)N(s)$$

$$D'(s) = r(s)(\phi(s) - 1)N(s)$$

$$N(s) = \sigma\left(\left(z - w(s) + (1 - \rho_x)Q(s)\right)K - D\right) + (1 - \sigma)\, w_e z$$

A.2 Matching $[Q, \theta, v, u, m, q, w]$

$$\theta(s) \equiv v(s)/u(s)$$

$$m(s) = \psi_0 u(s)^{\psi_1} v(s)^{1-\psi_1} = \psi_0\theta(s)^{1-\psi_1} u$$

$$q(s) = m(s)/v(s) = \psi_0\theta(s)^{-\psi_1}$$

$$u(s) = 1 - K$$

$$Q(s) = \frac{\kappa z}{q(\theta(s))} = \frac{\kappa z}{\psi_0}\theta(s)^{\psi_1}$$

$$K'(s) = m(s) + (1 - \rho_x)K$$

$$w(s) = \bar{w}z^{\gamma}$$

A.3 Goods $[r, c]$

$$r(s) = \frac{u'(c(s))}{\beta\mathrm{E}[u'(c(s'))|s]}$$

$$zK = c(s) + \kappa z v(s) = c(s) + \kappa z\theta(s)u(s)$$

B Steady state equations, incomplete markets model

B.1 Experts $[\phi, \nu, r_k, \lambda, K, D, N]$

$$\nu = \Lambda\phi \tag{37}$$

$$\nu = \frac{\beta (1-\sigma) ((r_k - r) \phi + r)}{1 - \beta\sigma ((r_k - r) \phi + r)} \tag{38}$$

$$r_k \equiv \frac{z - w(z)}{Q} + 1 - \rho_x$$

$$Q = \frac{\beta (1 - \sigma + \sigma\nu) (z - w + (1 - \rho_x) Q)}{\beta (1 - \sigma + \sigma\nu) r + \frac{\lambda\Lambda}{1+\lambda}}$$

$$QK = \phi N$$

$$D = r(\phi - 1)N$$

$$N = \sigma ((z - w(z) + (1 - \rho_x) Q) K - D) + (1 - \sigma) w_e z$$

B.2 Matching $[Q, \theta, v, u, m, q, w]$

$$\theta \equiv v/u$$

$$m = \psi_0 \theta^{1-\psi_1} (1 - K)$$

$$q = m/v = \psi_0 \theta^{-\psi_1}$$

$$K + u = 1$$

$$Q = \frac{\kappa z}{\psi_0} \theta^{\psi_1}$$

$$m = \rho_x K$$

$$w = \bar{w}$$

B.3 Goods $[r, c]$

$$r = 1/\beta$$

$$zK = c + \kappa z \theta (1 - K)$$

C Proofs

Proof of Lemma 1. We can set up the lagrangian:

$$\begin{aligned}\mathcal{L} = &\mathrm{E}\left[\Omega(s',s)\left((1-\sigma)n' + \sigma V(n',s')\right)\middle| s\right] \\ &+ \lambda\left(\mathrm{E}\left[\Omega(s',s)\left((1-\sigma)n' + \sigma V(n',s')\right)\middle| s\right] - \Lambda\left(Q(s)k'_o + \frac{\kappa z}{q(\theta)}k'_n\right)\right) \\ &+ \mu k'_n\end{aligned} \tag{39}$$

Where it is understood that any n' are replaced using (4). The FOCs wrt k'_n and k'_o are:

$$\frac{\partial \mathcal{L}}{\partial k'_o} = (1+\lambda)\mathrm{E}\left[\Omega(s',s)\left((1-\sigma)+\sigma V_1(n',s')\right)\left(z' - w(s') + (1-\rho_x)Q(s') - r(s)Q(s)\right)\middle| s\right] - \lambda\Lambda Q(s) = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial k'_n} = (1+\lambda)\mathrm{E}\left[\Omega(s',s)\left((1-\sigma)+\sigma V_1(n',s')\right)\left(z' - w(s') + (1-\rho_x)Q(s') - r(s)\frac{\kappa z}{q(\theta)}\right)\middle| s\right] - \lambda\Lambda\frac{\kappa z}{q(\theta)} + \mu = 0 \tag{41}$$

Defining $\tilde{\Omega} \equiv \Omega(s',s)\left((1-\sigma)+\sigma V_1(n',s')\right)$ and rearranging:

$$\frac{\partial \mathcal{L}}{\partial k'_o} \Rightarrow Q(s) = \frac{(1+\lambda)\mathrm{E}\left[\tilde{\Omega}\left(z' - w(s') + (1-\rho_x)Q(s')\right)\middle| s\right]}{(1+\lambda)\mathrm{E}\left[\tilde{\Omega}\middle| s\right]r(s) + \lambda\Lambda} \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial k'_n} \Rightarrow \frac{\kappa z}{q(\theta)} = \frac{(1+\lambda)\mathrm{E}\left[\tilde{\Omega}\left(z' - w(s') + (1-\rho_x)Q(s')\right)\middle| s\right] + \mu}{(1+\lambda)\mathrm{E}\left[\tilde{\Omega}\middle| s\right]r(s) + \lambda\Lambda} \tag{43}$$

From this we see that if $\mu = 0$ then the FOCs require that:

$$Q(s) = \frac{\kappa z}{q\left(\theta(s)\right)} \tag{44}$$

Since μ was the multiplier on the non-negative vacancies constraint, this means that if experts are happy to post vacancies in equilibrium, then $Q = \kappa z/q$. Since aggregate vacancies are typically positive in the data I'll restrict attention to the region where $\mu = 0$. This allows us to impose the condition $Q = \kappa z/q$ and treat existing matches and vacancies as the same from the expert's point of view. Defining $k' \equiv k'_o + k'_n$, we can re-express (4) as

$$n' = \left(r_k(s',s) - r(s)\right)Q(s)k' + r(s)n \tag{45}$$

Where $r_k(s',s) \equiv (z' - w(s') + (1-\rho_x)Q(s'))/Q(s)$. The limited commitment constraint (5) becomes

$$\Lambda Q(s)k' \leq V^{\diamond}(n,s;k') \tag{46}$$

Where the new conditional value function is:

$$V^{\diamond}(n,s;k') = \mathrm{E}\left[\Omega(s',s)\left((1-\sigma)n' + \sigma V(n',s')\right)\middle| s\right] \tag{47}$$

With n' replaced with the value implied by (45). Experts maximise value:

$$V(n,s) = \max_{k'} V^{\diamond}(n,s;k') \tag{48}$$

Subject to (45) and (46). I ignore the $v \geq 0$ constraint since I have assumed it is not binding. It is possible to show that the expert's problem is linear in n, which allows us to aggregate. If this is true the conditional value function is given by:

$$V^{\diamond}(n,s;k') = \mathrm{E}\left[\Omega(s',s)\left(1-\sigma+\sigma\nu(s')\right)n' \middle| s\right] \tag{49}$$

Where $V(n,s) = \nu(s)n$. We can define $\phi \equiv Q(s)k'/n$ and rewrite the flow BC as:

$$n' = \left(\left(r_k(s',s) - r(s)\right)\phi + r(s)\right)n \tag{50}$$

Thus n' is a function only of ϕ and n, and not k'. Hence we can rewrite the conditional value function as $\nu(s;\phi)n = V^{\diamond}(n,s;k')$:

$$\nu(s;\phi)n = \mathrm{E}\left[\Omega(s',s)\left(1-\sigma+\sigma\nu(s')\right)n' \middle| s\right] \tag{51}$$

Hence the overall maximisation can be written as

$$\nu(s)n = \max_{\phi} \nu(s;\phi)n \tag{52}$$

Subject to (50) and to

$$\Lambda Q(s)k' \leq \nu(s;\phi)n \Rightarrow \Lambda\phi \leq \nu(s;\phi) \tag{53}$$

This gives us the policy and value functions:

$$Q(s)k(s,n) = \phi(s)n$$

$$d(s,n) = (\phi(s)-1)n$$

$$V(n,s) = \nu(s)n$$

□

Proof of Lemma 2. Setting up the lagrangian:

$$L(n,s) = (1+\lambda)\mathrm{E}\left[\Omega(s',s)\left((1-\sigma)n' + \sigma V(n',s')\right)\middle| s\right] + \mu\left(n + \int_{s'} d(s')\,\mathrm{d}s' - Q(s)k'\right) - \lambda\Lambda Q(s)k' \tag{54}$$

Where I am implicitly assuming that vacancies are positive, λ is the multiplier on the moral hazard constraint, μ is the multiplier on the balance sheet, and it is understood that all n' are replaced using (28). The first order condition with respect to a generic $d(s')$ gives:

$$(1+\lambda)p(s'|s)\Omega(s',s)\left(1-\sigma+\sigma V_1(n',s')\right)r(s') = \mu \tag{55}$$

Using the consumer's first order condition, (29), to remove $r(s')$:

$$(1+\lambda)\left(1-\sigma+\sigma V_1(n',s')\right)=\mu \tag{56}$$

Since λ and μ are common for all s', this shows that the expert chooses state contingent debt to equalise the marginal value of net worth, $V_1(n',s')$ across states next period. It is possible (proof omitted) to prove that as usual expert value is linear in net worth: $V(n,s)=\nu(s)n$, which implies that $V_1(n,s)=\nu(s)$. Combining this with (56), this implies that the value of $\nu(s')$ doesn't depend on next period's shock. Using this, we can guess and verify an equilibrium where $\nu(s)$ is constant across states and time: $\nu(s)=\bar{\nu}\ \forall s$. Notice that if the moral hazard constraint is always binding then this implies that leverage is also constant: $\bar{\phi}=\bar{\nu}/\Lambda$.

We can actually completely characterise asset prices and the labour market using the recursive definition of expert value, and the fact the ν and ϕ are constant. Using (51), (27) and (28):

$$\bar{\nu}=\mathrm{E}\left[\Omega(s',s)\left(1-\sigma+\sigma\bar{\nu}\right)\left(r_k(s',s)\bar{\phi}+(1-\bar{\phi})r(s')\right)\Big|\, s\right] \tag{57}$$

Using (29) to remove $r(s')$ leaves:

$$\bar{\nu}=\left(1-\sigma+\sigma\bar{\nu}\right)\mathrm{E}\left[\Omega(s',s)r_k(s',s)\big|\, s\right]\bar{\phi}+(1-\bar{\phi})\left(1-\sigma+\sigma\bar{\nu}\right) \tag{58}$$

Or:

$$\mathrm{E}\left[\Omega(s',s)r_k(s',s)\big|\, s\right]=\frac{\bar{\nu}-(1-\bar{\phi})\left(1-\sigma+\sigma\bar{\nu}\right)}{\left(1-\sigma+\sigma\bar{\nu}\right)\bar{\phi}} \tag{59}$$

Notice that this is exactly the same as the definition of job value in the standard economy, except for the term on the right hand side, which would be one in that case. Notice as well that the term on the right hand side is constant over states and time, hence the interpretation is that the complete markets model is similar to the model without financial frictions, apart from a steady state "wedge".

It is easy to verify that the other equilibrium conditions are satisfied, verifying our guess. The model only has two state variables, z and K, as in the standard model. Net worth is not a state variable because of the state contingent contracts. Net worth is calculated as the required net worth for experts to purchase the capital stock, and we back out the required past debt choice each period to make this hold. □

Proof of Proposition 1. $R(Q)=\phi N$. I prove that $R(Q)$ is decreasing in Q by showing that both ϕ and N are. First note that Q only affects ϕ and N via r_k, and that r_k is decreasing in Q. ϕ is increasing in r_k because higher r_k increases expert value, allowing higher ϕ via $\phi=\nu/\Lambda$.[10] Hence ϕ is decreasing in Q. Steady state N is given by:

$$N=\frac{(1-\sigma)w_e z}{1-\sigma(r_k-r)\phi-\sigma r}$$

[10] We can show that ϕ is increasing r_k by considering perturbations to (37) and (38). Increasing r_k increases ν in (38), which allows higher ϕ in (37) which feeds back into higher ν in (38) and so on.

Steady state net worth is increasing in r_k as long as $N > 0$, and increasing in ϕ as long as $r_k > r$, which is required in a steady state where the financial friction binds. Since ϕ is also increasing in r_k, N is increasing in r_k and hence decreasing in Q. Therefore, $R(Q)$ is decreasing in Q. $\square$

Proof of Proposition 3. The model equations are satisfied by constant values for u, θ, m, q, and K, and values for c, w, and Q which are proportional to z. In particular, the free entry condition

$$Q(s) = \frac{\kappa z}{\psi_0}\theta(s)^{\psi_1}$$

is satisfied by constant θ and $Q = \bar{Q}z$. The recursive value of a match, with the definition of r_k plugged in, becomes

$$\mathrm{E}\left[\beta\frac{c(s)}{c(s')}\frac{z' - \bar{w}z' + (1-\rho_x)Q(s')}{Q(s)}\middle| s\right] = 1$$

which is satisfied by $Q(s) = \bar{Q}z$ and $c = \bar{c}z$. $\square$

Proof of Proposition 4. The model equations are satisfied by constant values for u, θ, m, q, ϕ, v and K, and values for c, w, and Q which are proportional to z. To see this, if we guess a constant value for ϕ then (13) implies a constant value for ν. We can verify that this satisfies the recursion for expert value with the definitions of Ω and r_k substituted in, and r replaced using (23):

$$\nu(s) = \mathrm{E}\left[\beta\frac{c(s)}{c(s')}\left(1-\sigma+\sigma\nu(s')\right)\left(\frac{z' - \bar{w}z' + (1-\rho_x)Q(s')}{Q(s)}\phi(s) + (1-\phi)\frac{c(s')}{\beta c(s)}\right)\middle| s\right]$$

This is satisfied for any z if ϕ and ν are constant and $Q(s) = \bar{Q}z$ and $c(s) = \bar{c}z$. If $\sigma = 0$ then (14) implies that N is proportional to z: $N(s) = w_e z$. This implies constant employment from (15):

$$K'(s) = \frac{\phi N(s)}{Q(s)}$$

Both N and Q are proportional to z, leaving K' constant. The value of D' at any time can be backed out from (16). $\square$

Proof of Proposition 5. In the proof of Lemma 2 I proved that there exists an equilibrium with constant leverage and expert value. I now show that under the assumptions of Proposition 5 the rest of the model equations can be satisfied with constant employment. As previously discussed, constant employment requires constant tightness, which implies that $Q(s) = \bar{Q}z$ via (8). This satisfies the main complete markets equation, (33) with the definitions of Λ and r_k substituted in:

$$\mathrm{E}\left[\beta\frac{c(s)}{c(s')}\frac{z' - \bar{w}z' + (1-\rho_x)Q(s')}{Q(s)}\middle| s\right] = \frac{\bar{\nu} - (1-\bar{\phi})\left(1-\sigma+\sigma\bar{\nu}\right)}{\left(1-\sigma+\sigma\bar{\nu}\right)\bar{\phi}}$$

This is satisfied with $Q(s) = \bar{Q}z$ and $c(s) = \bar{c}z$. (15) requires that $N(s) = \bar{N}z$, which is always feasible for any path of shocks by picking the right sequence of contingent claims. $\square$

D Alternative setup: competitive match producing firms

The model in the paper is equivalent to a model where experts only trade in completed matches, and there exists a perfectly competitive "match producing sector". The match producing sector pays vacancy posting costs and sells any completed matches on the spot market to experts. The match producing sector's problem is thus a static profit maximisation problem. Profit is given by:

$$\pi = Q(s)k_n' - \kappa z v$$

Where $k_n' = q(\theta(s))v$ is the number of successful matches a match producing firm produces if it posts v vacancies. Plugging this in and taking the FOC with respect to v (or imposing zero profit) gives us the arbitrage equation from the main model, $Q(s) = \frac{\kappa z}{q(\theta(s))}$. The boundary case with no vacancy posting is also supported here, since match producing firms must produce positive vacancies.

E Figures and tables

Table 3: Calibration

	Interpretation	Value	Source
β	Discount factor	0.9966	4.17% annual interest rate
σ_c	Risk aversion	1	Standard value
ρ_x	Job destruction	0.0274	Steady state transition probabilities
ψ_0	Match efficiency	0.3917	Steady state transition probabilities
ψ_1	Matching elasticity	0.5	Petrongolo & Pissarides (2001)
κ	Recruiting cost	0.3107	0.32 $\times$ steady state wage
$\bar{w}$	Steady state real wage	0.9709	Steady state unemployment
γ	Real wage flexibility	0.7	Michaillat (2012)
σ	Expert exit prob.	0.9770	Asset price moments
w_e	New expert equity	0.3026	Asset price moments
Λ	Fraction of divertable capital	0.4854	Asset price moments
$\bar{z}$	Steady state productivity	1	Normalisation
ρ	Autocorrelation of productivity	0.98975	Quarterly (log HP-filtered) autocorrelation
σ_e	Standard deviation of ε	0.0043	Quarterly (log HP-filtered) std.

Table 4: Simulated moments of the model without financial frictions

	u	v	θ	w	y	z	Q/z
Standard deviation	0.104	0.127	0.226	0.010	0.021	0.015	0.113
	(0.016)	(0.017)	(0.034)	(0.002)	(0.003)	(0.002)	(0.017)
Autocorrelation	0.936	0.831	0.913	0.907	0.927	0.907	0.913
	(0.018)	(0.045)	(0.025)	(0.026)	(0.021)	(0.026)	(0.025)
Correlation	1	-0.886	-0.965	-0.959	-0.980	-0.959	-0.965
		(0.007)	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)
	–	1	0.977	0.981	0.961	0.981	0.977
			(0.001)	(0.001)	(0.002)	(0.001)	(0.001)
	–	–	1	1.000	0.998	1.000	1.000
				($1e^{-5}$)	($1e^{-4}$)	($1e^{-5}$)	(0)
	–	–	–	1	0.996	1.000	1.000
					($2e^{-4}$)	($1e^{-16}$)	($1e^{-5}$)
	–	–	–	–	1	0.996	0.998
						($2e^{-4}$)	($1e^{-4}$)
	–	–	–	–	–	1	1.000
							($1e^{-5}$)
	–	–	–	–	–	–	1

I simulate the model 100 times for 526 months, corresponding to the length of the data sample. I convert the data to quarterly frequency as per Michaillat (2012). All data are quarterly, logged and then HP-filtered with smoothing parameter 10^5. *The numbers in parentheses are the standard errors of the sample statistics.*