Fiscal Policy with Limited-Time Commitment

Alex Clymo[§] Andrea Lanteri[¶]

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Abstract

We consider models where the Ramsey-optimal fiscal policy under Full Commitment (FC) is time-inconsistent and define a new notion of optimal policy, Limited-Time Commitment (LTC). Successive one-period lived governments can commit to future plans over a finite horizon. We provide a sufficient condition on the mapping from finite policy sequences to allocations, such that LTC and FC lead to the same outcomes. We then show that this condition is verified in several existing models, allowing FC Ramsey plans to be supported with a finite commitment horizon (often a single period). We relate the required degree of commitment to the economic environment: in economies without capital, the minimum degree of commitment required is given by the government debt maturity; in economies with capital and government balanced-budget constraints, the required commitment is given by the horizon over which the budget has to be balanced. Finally, we solve numerically for the LTC equilibrium of an economy where the equivalence result fails and show that a single year of commitment to capital taxes provides substantial welfare gains relative to the No-Commitment time-consistent policy.

Keywords: Optimal fiscal policy; time-inconsistency; limited commitment.

[§]Faculty of Economics and Business, University of Amsterdam. Email: `a.j.clymo@uva.nl`.

[¶]Department of Economics, Duke University. Email: `andrea.lanteri@duke.edu`.

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1 Introduction

Governments in advanced economies tend to formulate their macroeconomic policies as plans for a finite future horizon. In particular, fiscal policy is typically decided upon and announced before or at the beginning of the fiscal year and remains fixed for the duration of the year.[1] Reforms such as VAT rate changes and fiscal consolidation plans are also announced before their implementation and typically contain details of short-to-medium run policy plans.[2] Moreover, the political process makes it hard to change contemporaneous policies, with the result that policy changes are often implemented with a delay.

In contrast, a large part of the literature on optimal fiscal policy assumes either that a single government at the beginning of time has Full Commitment (FC) into the infinite future, or that in each period there is a government with No Commitment (NC) at all, only able to choose contemporaneous policies. On the face of it, both of these extreme assumptions appear hard to reconcile with the fact that policymakers act and communicate as if they possessed a limited degree of commitment over a finite future horizon. On the one hand, policymakers are only in power for a limited period of time, which makes commitment into the infinite future impossible. On the other hand, institutional features and political delays may place limits on a policymaker's ability to change contemporaneous policy instruments, while allowing for the possibility of planning policy changes for a near future horizon.

Motivated by this apparent distance between observation and theory, in this paper we study fiscal policy when successive benevolent governments inherit the plans of their predecessors and formulate plans for a finite future horizon. In this formulation, which we call Limited-Time Commitment (LTC), governments cannot commit into the infinite future, but instead only possess the ability to commit for a finite number of periods. Specifically, we define governments as having L periods of commitment if the time-t government cannot change policies dated time t to $t+L-1$, and chooses policies dated time $t+L$. LTC thus lies between FC and NC, and we ask the natural

[1]For example, in the UK the budget is announced on "Budget Day" in March, before the start of the fiscal year, and typically passed shortly afterwards.

[2]Examples of significant pre-announced VAT reforms include Japan (announced in 1996, implemented in 1997) and Germany (announced in 2005, implemented in 2007). Alesina et al. (2015) document many examples of multi-year fiscal consolidations.

question of whether governments with only a finite number of periods of commitment behave more like governments with full, or no, commitment.

Our main theoretical contribution is an equivalence result: we provide a set of sufficient conditions under which LTC sustains FC outcomes. Under these conditions, we apply the same Markov perfect equilibrium (MPE) concept introduced by the literature on time-consistent policies to our setup and show that a limited number of periods (often a single period) of future commitment is sufficient to sustain the same allocations and policies that arise when a single time-0 government can commit into the infinite future.

The time-inconsistency problems we study arise from governments formulating optimal policies subject to competitive-equilibrium constraints which contain future values of certain choice variables. We formulate a general deterministic framework in which we establish the conditions under which LTC can sustain the FC policy. If a *finite* sequence of future policy instruments is sufficient to uniquely pin down a certain finite sequence of contemporaneous and future allocations, then the time-inconsistency problem can be resolved if the government can commit to a finite sequence of policies of a sufficient length.

To build intuition in a simple case, consider a government elected at time t that chooses taxes in order to maximize welfare subject to competitive equilibrium conditions, including an Euler equation for government bonds, which contains marginal utility, and hence consumption, at $t+1$. The time-inconsistency problem here is that the time-$t+1$ government, who no longer has to respect the time-t Euler equation, could choose policies leading to a different consumption allocation from the time-t promise. However, if time-$t+1$ consumption is fully pinned down by the time-$t+1$ policy instruments, then the time-t government can prevent the time-$t+1$ government from deviating from her promise if she can commit to taxes one period ahead. The time-$t+2$ government will also respect the promises of the time-$t+1$ government, creating a chain of commitment which is able to sustain the FC policy.

After formalizing this intuition in our general framework, we show that the key condition for equivalence of LTC and FC is indeed satisfied in many models of optimal fiscal policy that have been proposed in the literature. This implies that even if assuming FC into the infinite future may seem extremely "unrealistic", depending on the model environment, it may lead to the same outcomes

that arise in a more empirically plausible setup where there is a succession of governments that make decisions for the near future. Importantly, we show how the minimum degree of commitment necessary to sustain FC outcomes in a given model depends on features of the economic environment, such as technology and asset market structure, and not on the arbitrary definition of the length of a period.

In models without capital, where governments choose the timing of labor taxes and government debt (e.g., Lucas and Stokey, 1983), the FC policy can be supported with commitment equal to the longest government bond maturity. An implication of this result is that commitment frictions are unlikely to play a large role in models with one-period bonds only, where a period is a quarter or a year. Models that emphasize lack of commitment to fiscal policy should instead pay close attention to matching the debt maturity structure.[3]

In models with capital, and governments who choose capital and labor taxes subject to a balanced budget constraint (a version of the models of Judd, 1985, Chamley, 1986, similar to the one introduced by Stockman, 2001), we show that FC can be supported with commitment equal to the length of time over which the government budget must be balanced (or to the length of time to build of capital, if it is longer). In particular, under the commonly used assumptions of a yearly calibration and balanced budgets every period (e.g. Klein et al., 2008), we show that the Chamley-Judd result of zero capital taxes in the long run can be recovered with a single period of commitment. This result stands in stark contrast to the outcomes obtained under NC, which typically feature high capital taxes even in the long run. However, without the balanced-budget assumption, LTC cannot support FC for any finite degree of commitment. Thus, our results uncover a role for balanced-budget rules in building commitment in environments where governments can only commit to a finite-horizon plan.

A further implication of our equivalence result is that, whenever LTC and FC lead to the same outcomes, it is possible to compute the time-inconsistent solution to the FC problem using standard dynamic programming tools. One can simply use the required pre-committed policies as state variables when "recursifying" the solution, and then apply standard numerical tools, such as value

[3] Consistent with this finding, Debortoli et al. (2016) analyze the optimal maturity structure of government debt in a model without commitment, and find large differences from the FC policy.

function iteration. Whenever applicable, our approach of including pre-committed policy variables as states constitutes an intuitive alternative to using, for example, promised utilities or Lagrange multipliers when solving the FC problem. We illustrate our algorithm for the Lucas and Stokey (1983) economy.

Finally, we investigate a model where our equivalence result does not hold. This is a version of the Klein et al. (2008) model of optimal public good provision funded by capital taxation, but with exogenous labor supply.[4] We show that a single year of commitment is sufficient to recover around one third of the welfare difference between FC and NC, since the LTC government internalizes the distortionary effect of one-year-ahead capital taxes. We then simulate the effects of a "constitutional reform", imposing one year of commitment to taxes and spending starting from the steady-state of an economy with NC. This reform involves a non-monotone path for the size of the government and leads to significant welfare gains (approximately 1.7% of permanent consumption), highlighting the importance of designing institutions that guarantee a limited degree of commitment in economies where the government lacks credibility.

Related Literature. The time-inconsistency of optimal policy has been a central issue in the macroeconomic literature since the 1970s. Kydland and Prescott (1977) highlighted that optimal policy in a dynamic model involves ex-ante promises that appear suboptimal ex-post if the government is allowed to reoptimize at a later date. For this reason, the presence of future (expected) variables in the constraint set of Ramsey-optimal plans rules out the use of standard optimal control techniques.

Despite its importance both in the academic and in the policy debate, this key insight has not led to a uniform reaction in the literature. A part of the optimal policy research agenda has worked on modifying standard recursive methods to deal with this class of problems under the assumption that the government is endowed with a Full Commitment technology into the infinite future, while another strand of the literature has considered time-inconsistency a central, unavoidable problem and has focused on equilibria with No Commitment instead.

As leading examples of the first approach, consider the Recursive Contracts method formalized

[4] We discuss how this modification breaks our equivalence result.

in Marcet and Marimon (2011) which adds the Lagrange multipliers on forward-looking constraints as state variables, allowing the reformulation of optimal policy problems as recursive saddle-point problems, or the Promised Utility approach proposed by Abreu et al. (1990), based on the result that past histories can be summarized by promised utility. Relatedly, Kydland and Prescott (1980) and Chang (1998) proposed similar method based on the addition of future marginal utilities as state variables in the optimal policy problem. In this paper we propose an alternative, intuitive, recursive method to solve for optimal policy with FC, based on using policies as state variables. This method works whenever our equivalence result holds.

The second part of the literature has focused instead on formulating equilibrium concepts without a commitment technology, starting with the seminal paper on Sustainable Plans by Chari and Kehoe (1990), and the application of Markov perfect NC equilibria in optimal policy games, as for instance in Klein and Ríos-Rull (2003), Krusell et al. (2004) and Klein et al. (2008). In these papers, there is a succession of governments that can only choose contemporaneous policy instruments. Hence, any ability to formulate credible promises about future allocations is ruled out by assumption.[5]

In our paper, we explore an intermediate assumption on the commitment technology, namely that a succession of governments can announce policies for a finite future horizon. We allow for some commitment to future outcomes as in the first strand, but we apply the Markov perfect equilibrium concept typical of the NC literature. We show that in several models that have been studied in the optimal fiscal policy literature (e.g. a deterministic version of Lucas and Stokey, 1983) a few periods of commitment, often only one, are sufficient to sustain the FC outcome as the unique equilibrium of our game.

Our results on the degree of commitment required to sustain FC outcomes in specific models are related to several strands of literature. In economies without capital, we show that the government debt maturity is a key variable: longer debt calls for a longer commitment horizon. Consistent with this finding, Debortoli et al. (2016) show that the welfare cost of lack of commitment is large and

[5] A partial exception is Klein and Ríos-Rull (2003): in their setup, the government can set capital taxes one period ahead, but only current labor taxes. They show that capital taxes are on average high, compared to the FC equilibrium which has average capital taxes near zero. In Section 4 we solve a deterministic version of the same model and show that if instead the time-t government chooses both the time-$t+1$ capital *and* labor taxes then LTC sustains the FC policy and allocation.

dominates the cost of lack of tax smoothing in presence of long-maturity bonds. In economies with capital, we uncover a role for balanced-budget rules in allowing to sustain FC plans with LTC. The previous literature, e.g. Halac and Yared (2014), has focused on the role of fiscal rules in overcoming governments' present bias. We show that a role for fiscal rules also arises when the government shares the same discount factor as households, and the source of time-inconsistency is the presence of forward-looking constraints.[6]

Close to our work, in terms of motivation, are the Quasi Commitment approach of Schaumburg and Tambalotti (2007), and the Loose Commitment approach formulated by Debortoli and Nunes (2010, 2013), which build on an early contribution by Roberds (1987). These papers assume that a government can formulate a plan into the infinite future, but with some probability in every future period a new government is elected and allowed to change the plan. Like ours, this game represents an intermediate point in the FC vs. NC debate. Differently from these papers, however, LTC gives the government full commitment within a limited, deterministic time horizon, instead of probabilistic commitment over an infinite horizon. While this may seem a technical difference, it turns out to be important. We show that in some models where LTC with one period of commitment leads to allocations equivalent to FC, Loose Commitment with *on average* one period of commitment leads to allocations closer to NC. In this sense a result of this paper is that once you limit the commitment technology of the government, the details of how you do so matter for the results.[7]

Another related strand of literature studies the so-called "timeless perspective" in optimal policy (e.g., Woodford, 2011). In this setup, the policymaker is constrained to choose current policies as

[6]Domeij and Klein (2005) study tax reform in presence of implementation lags. This approach leads to a similar timing assumption to ours, but with an important distinction: under our main assumption, every government is inheriting the "right" policy from its predecessor, hence FC policies can be sustained in equilibrium, whereas in Domeij and Klein (2005) the focus is on a fiscal reform starting from a suboptimal policy. Martin (2015) studies the effects of within-period commitment to fiscal policy. In contrast, we analyze the effects of commitment across a finite number of periods.

[7]An alternative approach is given in the reputational equilibria literature, starting with the seminal contribution of Barro and Gordon (1983). Bassetto (2016) distinguishes the communication and implementation of future policies, and shows that there is no separate role for communication if the government has the same information as the private sector. Other papers explore the extent to which FC outcomes can be supported by adding extra policy instruments, e.g. Alvarez et al. (2004), Conesa and Dominguez (2012) and Laczó and Rossi (2015).

if they were part of an optimal FC plan chosen in an infinitely far past. An important difference between this approach and ours it that with LTC, under the "right" initial conditions, we recover exactly the FC plan, whereas the optimal "timeless" policy would lead to a different outcome, because it is designed to remove the discrepancy in the FC plan between time-0 policies and the rest of the plan.[8]

The rest of the paper is organized as follows. Section 2 introduces the concept of LTC in a general model and discusses informally its equivalence with FC in a simple example. Section 3 proves formally the main equivalence result. Section 4 applies the result to models of optimal fiscal policy commonly used in the literature and discusses the role of initial policy conditions. Section 5 shows how to solve for FC optimal policy using taxes as state variables. Section 6 discusses a model where our equivalence result fails and provides numerical results. Section 7 concludes.

2 Full Commitment and Limited-Time Commitment

In this section we describe a generic model economy, and define two notions of optimal policy: Full-Commitment Ramsey equilibrium and Limited-Time Commitment equilibrium. We argue that LTC nests both the NC equilibrium, as a special case without any degree of commitment, and the FC equilibrium, in the limit with infinite commitment, under the right initial conditions.

For each of these equilibrium concepts, we also discuss the specific assumptions on the mapping from *infinite* sequences of policies to *infinite* sequences of allocations needed for the government(s) to be able to uniquely pin down competitive equilibrium for a given policy. This allows the statement of the optimal policy problem in each case.

We then describe a simple example and use it to provide intuition for our main result: under a stronger assumption on the mapping from *finite* sequences of policies to *finite* sequences of allocations, LTC with a sufficiently long, but finite, commitment, and FC give rise to equivalent

[8] To make a concrete example, consider a deterministic version of a model of the optimal timing of taxes (Lucas and Stokey, 1983). The FC plan calls for a constant tax rate from period 1 onwards, and a different tax rate in period 0. With one period of commitment, LTC sustains the same outcome. The timeless perspective would call for a constant tax rate from period 0, and, importantly, this tax rate is different from both the time-0 and the time-1 tax rate arising under FC.

outcomes. This result is shown formally in Section 3, and applied to several models of optimal policy in Section 4.

2.1 Environment and competitive equilibrium

Time is discrete and indexed by $t = 0, 1, 2, ...$ The economy is populated by a continuum of households, a continuum of firms, and a government or a sequence of governments. A vector of exogenous variables $g_t \in G$ follows a deterministic sequence about which all agents have perfect foresight, with the Markov property that g_t is sufficient information to predict g_{t+1}. Denote the transition for g_t by $\Gamma^g : G \mapsto G$, such that $g_{t+1} = \Gamma^g(g_t)$. We call $b_t \in B$ the endogenous state variables (b_0 being an initial condition), $c_t \in C$ the remaining variables constituting allocations (for instance consumption and hours worked), $p_t \in P \subset \mathbb{R}^{n_P}$ the prices and $\tau_t \in T$ the policy instruments chosen by the government(s).

Households' preferences are represented by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t r(c_t, b_t, g_t, \tau_t) \tag{1}$$

where $r : C \times B \times G \times T \mapsto \mathbb{R}$ and $\beta \in (0, 1)$. Households and firms take sequences of prices $\{p_t\}_{t=0}^{\infty}$ and policy instruments $\{\tau_t\}_{t=0}^{\infty}$ as given. Households maximize utility subject to their budget constraints (and potentially other constraints such as borrowing constraints). Firms maximize profits subject to their production technologies. All markets clear.

Following the general formulation in Marcet and Marimon (2011), we can summarize these equilibrium conditions with three sequences of constraints: a transition equation for the endogenous states, a set of constraints involving only contemporaneous allocations and a set of constraints involving future variables.

$$b_{t+1} = l(b_t, g_t, c_t, p_t, \tau_t) \tag{2}$$

$$k(b_t, g_t, c_t, p_t, \tau_t) \leq 0 \tag{3}$$

$$h(b_t, g_t, c_t, p_t, \tau_t, ..., b_{t+N}, g_{t+N}, c_{t+N}, p_{t+N}, \tau_{t+N}) = 0. \tag{4}$$

for $N \geq 1$.

Definition 1. *Given an initial condition* b_0*, an exogenous sequence* $\{g_t\}_{t=0}^{\infty}$ *and a policy sequence* $\{\tau_t\}_{t=0}^{\infty}$*, a **competitive equilibrium** is a sequence* $\{c_t, p_t, b_{t+1}\}_{t=0}^{\infty}$ *that satisfies (2), (3) and (4) for* $t = 0, 1, ...$

As is well known in the literature, the presence of the future variables in the constraint set defining competitive equilibria is the reason for the time-inconsistency of FC policies. In Definition 2 we explicitly label the future variables which enter into the constraints as "problematic".

Definition 2. *Split* c_t *into its elements* $(c_t^1, ..., c_t^{n_c})$*. For every* $1 \leq s \leq N$*, we call **problematic** from the perspective of time* t *the elements of* c_{t+s} *which appear in the constraint (4). The same definition applies to elements of* b_{t+s} *(with* $s > 1$*),* p_{t+s} *and* τ_{t+s}*.*

We make the following (standard) regularity assumption.

Assumption 1. *Given an initial condition* b_0 *and an exogenous sequence* $\{g_t\}_{t=0}^{\infty}$*, for all sequences* $\{\tau_t, c_t, b_{t+1}, p_t\}_{t=0}^{\infty}$ *satisfying (2), (3), (4),* $\lim_{n\to\infty} \sum_{t=0}^{n} \beta^t r(c_t, b_t, g_t, \tau_t)$ *exists, although it may be plus or minus infinity.*

2.2 Full-Commitment Ramsey equilibrium

In a FC equilibrium, a single benevolent infinitely-lived government endowed with the ability to credibly commit into the infinite future announces a plan at $t = 0$ and then implements it. In order for the government to be able to pin down a unique competitive equilibrium, the following assumption on the mapping from infinite sequences of taxes to allocations is required.

Assumption 2. *Given an initial condition* b_0*, an exogenous sequence* $\{g_t\}_{t=0}^{\infty}$ *and a policy sequence* $\{\tau_t\}_{t=0}^{\infty}$*, there exists a unique sequence* $\{(c_t, b_{t+1}, p_t)\}_{t=0}^{\infty}$ *that satisfies equations (2), (3) and (4) for* $t = 0, 1, ...$

This assumption allows us to map an infinite sequence of policy instruments to a single competitive equilibrium, ensuring that the government knows exactly which equilibrium is selected for a given policy choice.[9] The FC government solves the following problem:

$$\max_{\{\tau_t, c_t, b_{t+1}, p_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t r(c_t, b_t, g_t, \tau_t) \tag{5}$$

[9] Bassetto (2005) argues that a full specification of out-of-equilibrium government strategies is necessary to guarantee that the desired equilibrium is obtained. In this paper we implicitly assume that there is a more general formulation

subject to (2), (3) and (4), initial condition b_0, and the exogenous sequence $\{g_t\}_{t=0}^{\infty}$.

Definition 3. *Fix an initial condition b_0, and an exogenous sequence $\{g_t\}_{t=0}^{\infty}$. Let $\left\{\tau_t^{FC}\right\}_{t=0}^{\infty}$ be a policy sequence that solves (5). A* ***Full-Commitment (FC) Ramsey equilibrium*** *is the competitive equilibrium associated with $\left\{\tau_t^{FC}\right\}_{t=0}^{\infty}$.*

As has been extensively discussed in the literature (e.g. Kydland-Prescott, 1977), the presence of problematic variables in (4) induces time-inconsistency in the FC policy. The FC government commits to a certain sequence which is optimal from a $t = 0$ viewpoint, but would no longer be optimal in subsequent periods, if a reoptimization were allowed.

2.3 Limited-Time Commitment equilibrium

The setup can be described as a game, where successive one-period-lived governments indexed by t choose only a finite sequence of policy instruments, taking as given the strategies of the following governments. Each government is benevolent and maximizes (1) subject to the competitive equilibrium conditions (2), (3) and (4). Such a game may often have multiple equilibria. In our exposition, we follow the literature (for instance Klein et al., 2008) in restricting attention to symmetric Markov perfect equilibria, where each government chooses a common best-response function mapping a small set of "natural" state variables into the chosen sequence of policy instruments.

Let $L = 0, 1, 2, ..$ index the duration of commitment. Let $S^0 \equiv \{1\}$ and $S^L \equiv \{1\} \cup T^L$ for $L > 0$ and let $\tau_t^L \in S^L$ be a vector of policies inherited from governments in power before t. In particular, $\tau_t^L \equiv 1$ for $L = 0$ and $\tau_t^L \equiv (1, \tau_t, \tau_{t+1}, ..., \tau_{t+L-1})$ for $L > 0$. In the case $L = 0$, no policy decision is inherited and LTC nests the NC equilibrium studied for instance by Klein et al. (2008). Hence, we let the corresponding (redundant) state variable be a constant number, which is equivalent to not having a policy-related state variable. Note that this should not be confused with the value of a policy instrument being equal to 1.

The government dated t takes as given the inherited policies τ_t^L and chooses the policy for period τ_{t+L}. In the NC game with $L = 0$, the government is free from policy constraints and chooses the value of the current policy instrument. In the simplest case of Limited-Time Commitment, with

of the government strategy, involving binding promises about out-of-equilibrium paths, that guarantees uniqueness of the desired Ramsey equilibrium.

$L = 1$ (which we label One-Period Commitment, or OPC), each government takes the current policy as given and chooses the next period policy. Relative to the NC case, a positive level of commitment $L > 0$ implies that the state vector describing the choice problem of a government has to be enriched to include all the policy instruments that have been chosen by previous governments and cannot be changed. Hence, the "natural" state variables of this problem are $\left(b_t, g_t, \tau_t^L\right)$ and accordingly we have to treat τ_0^L as an initial condition (as well as b_0).

Consider the government in power in period t and let $\tilde{V}^{LTC}(b_{t+1}, g_{t+1}, \tau_{t+1}^L)$ be the agent's tail of discounted utility starting in $t+1$ if the next government's state variables are $\left(b_{t+1}, g_{t+1}, \tau_{t+1}^L\right)$ and all governments from $t+1$ onwards are expected to play a common policy function $\tau_{t+j+L} = \tilde{\tau}^{NC}(b_{t+j}, g_{t+j}, \tau_{t+j}^L)$ for $j = 1, 2, ...$ In order to define $\tilde{V}^{LTC}(b_{t+1}, g_{t+1}, \tau_{t+1}^L)$ more formally, we restrict the governments to only playing policy functions for which the following assumption holds.

Assumption 2-LTC. *Given a state $(b_t, g_t, \tau_t^L) \in B \times G \times S^L$, a function $\tilde{\tau}^{LTC} : B \times G \times S^L \mapsto T :$ such that $\tau_{t+j+L} = \tilde{\tau}^{LTC}(b_{t+j}, g_{t+j}, \tau_{t+j}^L)$ for $j = 1, 2, ...$ and a time $t+L$ policy $\tau_{t+L} \in T$, the competitive equilibrium system given by (2), (3) and (4) for $s = t, t+1, ...$ admits a unique time-invariant solution for the vector (c_t, p_t, b_{t+1}) given by $(c_t, p_t, b_{t+1}) = \phi^{LTC}\left(b_t, g_t, \tau_t^L, \tau_{t+L}; \tilde{\tau}^{LTC}\right)$.*

This assumption ensures that given any state, and any policies being played by future governments, the choice of τ_{t+L} by the time-t government pins down a unique competitive equilibrium. When all governments play the policy function $\tilde{\tau}^{LTC}$, the function $\tilde{V}^{LTC}$ satisfies the following functional equation.

$$\begin{aligned} \tilde{V}^{LTC}\left(b_t, g_t, \tau_t^L\right) = r\left(\phi_1^{LTC}\left(b_t, g_t, \tau_t^L, \tau_{t+L}; \tilde{\tau}^{LTC}\right), b_t, g_t, \tau_t\right) \\ + \beta \tilde{V}^{LTC}\left(\phi_3^{LTC}\left(b_t, g_t, \tau_t^L, \tau_{t+L}; \tilde{\tau}^{LTC}\right), g_{t+1}, \tau_{t+1}^L\right) \end{aligned} \tag{6}$$

where $\tau_{t+L} = \tilde{\tau}^{LTC}\left(b_t, g_t, \tau_t^L\right)$ and ϕ_i^{LTC} is the i-th entry of the vector (c_t, p_t, b_{t+1}) implied by the competitive equilibrium mapping ϕ^{LTC}.

The government dated t anticipates that all future governments will play the policy function $\tilde{\tau}^{LTC}$ and solves the following maximization problem

$$\max_{\tau_{t+L}, c_t, p_t, b_{t+1}} r(c_t, b_t, g_t, \tau_t) + \beta \tilde{V}^{LTC}\left(b_{t+1}, g_{t+1}, \tau_{t+1}^L\right) \tag{7}$$

subject to $(c_t, p_t, b_{t+1}) = \phi^{LTC}\left(b_t, g_t, \tau_t^L, \tau_{t+L}; \tilde{\tau}^{LTC}\right)$, where future variables appearing in constraint (4), such as c_{t+1}, are given by $\phi_1\left(b_{t+1}, g_{t+1}, \tau_{t+1}^L, \tau_{t+L+1}; \tilde{\tau}^{LTC}\right)$, and so on.

In a symmetric Markov perfect equilibrium, the solution for the optimal policy instrument is given by $\tau_{t+L} = \tilde{\tau}^{LTC}\left(b_t, g_t, \tau_t^L\right)$ and the maximized value is $\tilde{V}^{LTC}\left(b_t, g_t, \tau_t^L\right)$. In words, the policy function $\tilde{\tau}^{LTC}$ is associated with a fixed point of the operator defined in (7).

Definition 4. *Fix initial conditions $\left(b_0, \tau_0^L\right)$, and an exogenous sequence $\{g_t\}_{t=0}^{\infty}$. A **symmetric Markov-perfect Limited-Time-Commitment (LTC) equilibrium** is a competitive equilibrium associated with the policy sequence $\left\{\tilde{\tau}^{LTC}\left(b_t, g_t, \tau_t^L\right)\right\}_{t=0}^{\infty}$ if $L = 0$ and with the sequence $\left(\tau_0, ..., \tau_{L-1}, \left\{\tilde{\tau}^{LTC}\left(b_t, g_t, \tau_t^L\right)\right\}_{t=0}^{\infty}\right)$ if $L > 0$.*

This definition includes the No Commitment equilibrium as the special case with $L = 0$. Notice that for $L \to \infty$, if we take as "initial conditions" the FC policy sequence then this equilibrium trivially coincides with the FC equilibrium, as in the limit the entire path of taxes coincides with the initial conditions. In this case, LTC nests both NC ($L = 0$) and FC ($L \to \infty$).

2.4 Example

We now discuss the relationship between the equilibrium notions of FC and LTC in the context of a simple deterministic version of the optimal fiscal policy model of Lucas and Stokey (1983). In this model, we informally argue that a single period of commitment ($L = 1$) is sufficient to sustain FC outcomes. To avoid notational clashes with the general framework presented above, here and wherever we refer to a specific model we use upright text to denote variables.

A representative household has preferences defined over sequences of private consumption $\{\mathrm{c}_t\}_{t=0}^{\infty}$ and labor effort $\{\mathrm{l}_t\}_{t=0}^{\infty}$:

$$\sum_{t=0}^{\infty} \beta^t u(\mathrm{c}_t, \mathrm{l}_t) \tag{8}$$

with standard assumptions $u_{\mathrm{c}} > 0$, $u_{\mathrm{cc}} < 0$, $u_{\mathrm{l}} < 0$, $u_{\mathrm{ll}} < 0$. Her budget constraint is given by[10]

$$\mathrm{c}_t + \mathrm{q}_t \mathrm{b}_{t+1} = \mathrm{w}_t \mathrm{l}_t \left(1 - \tau_t^l\right) + \mathrm{b}_t \tag{9}$$

where q_t is the price of a one-period discount bond, b_{t+1}, issued at t that repays one unit of consumption at $t+1$. w_t is the wage, and τ_t^l the proportional tax on labor income. The resource

[10] Here and throughout the paper we ignore the possibility that the government can issue lump-sum rebates. This is without loss of generality, and all results go through if the rebate is treated as a policy instrument, symmetrically with the other taxes.

constraint reads

$$\mathrm{c}_t + \mathrm{g} = \mathrm{l}_t \tag{10}$$

where g is a constant level of government expenditure that needs to be financed through the proportional labor income tax. Output is produced using a linear technology in labor: $y_t = l_t$, hence firms' profit maximization implies a unit wage: $\mathrm{w}_t = 1$. The government's budget is implied by the agent's budget constraint and the resource constraint:

$$\mathrm{b}_t + \mathrm{g} = \tau_t^l \mathrm{l}_t + \mathrm{q}_t \mathrm{b}_{t+1}. \tag{11}$$

The agent's first order conditions with respect to consumption, labor effort and bonds, together with the resource constraint, can be summarized by an intratemporal optimality condition and an Euler equation.

$$-\frac{u_{\mathrm{l}}(\mathrm{c}_t, \mathrm{c}_t + \mathrm{g})}{u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{c}_t + \mathrm{g})} = 1 - \tau_t^l \tag{12}$$

$$\mathrm{q}_t u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{c}_t + \mathrm{g}) = \beta u_{\mathrm{c}}(\mathrm{c}_{t+1}, \mathrm{c}_{t+1} + \mathrm{g}) \tag{13}$$

In terms of the general notation introduced above, we have $b_t = \mathrm{b}_t$, no exogenous state (g is constant), $c_t = (\mathrm{c}_t, \mathrm{l}_t)$, $p_t = \mathrm{q}_t$, and $\tau_t = \tau_t^l$. Note that in principle we should distinguish between the demand for bonds coming from household and the supply coming from the government (a policy variable), but for simplicity we directly impose bonds market clearing and refer to bonds generically as b_t. We have also implicitly solved out for the real wage. Equation (9) is an example of a constraint like (2), while (10) and (12) are examples of constraints like (3). Equation (13) represents a constraint like (4), involving c_{t+1} as the only "problematic" variable, after we have substituted out l_{t+1} using the resource constraint.

We refer to Lucas and Stokey (1983) for a detailed treatment of the FC optimal policy in this setup. When $t = 0$, the FC government has an incentive to use the initial allocation to twist the interest rate and decrease the value of outstanding initial debt b_0, hence reducing the distortions required to finance expenditure. In particular, if the FC government starts with a positive stock of debt, $\mathrm{b}_0 > 0$, she will choose a lower tax rate in the first period than in the later periods.

In this model, the source of time-inconsistency of the FC policy is the presence of c_{t+1} in the Euler equation for bonds. Only c_0 does not enter into an Euler equation from the previous period. Hence, any FC promises about allocations c_t with $t > 0$ would be reneged if the government was

allowed to reoptimize at t: in order to decrease the value of outstanding debt, b_t, the government would like to offer another tax cut, although she promised not to do this initially.

These are precisely the incentives faced by governments in the NC game, for whom outstanding debt b_t is the only "natural" state variable. Each government chooses the contemporaneous tax rate τ_t^l as a function only of b_t. The government in power in each period now has an incentive to twist the interest rate, leading to a deviation from the FC outcomes. The solution to the NC game in this and similar models is investigated by Krusell et al. (2004) and Debortoli and Nunes (2013).

Assume, however, that there is a positive, albeit finite, degree of commitment. In particular, consider the LTC game with $L = 1$. Each government takes the contemporaneous tax rate as given and chooses the policy for the following period. The "natural" state variables are (b_t, τ_t^l). Notice that now the government dated t is unable to affect the allocation $(\mathrm{c}_t, \mathrm{l}_t)$, which is entirely pinned down by (10) and (12), given the inherited tax rate τ_t^l. This implies that the government cannot twist the interest rate and affect the value of outstanding debt. Furthermore, by announcing (and committing to) a future tax rate τ_{t+1}^l, the government is effectively choosing (q_t, b_{t+1}) and, importantly, the "problematic" variable c_{t+1}, as well as l_{t+1} from the resource constraint at $t+1$. Hence, in this model, commitment to a finite sequence of policies, specifically a single future tax rate, is sufficient to uniquely pin down the future variables generating the time-inconsistency, thereby eliminating the ability of the future government to act in a way that is inconsistent with the FC plan. By doing so, starting from an initial condition consistent with the FC plan, a chain of successive governments with LTC will sustain the whole FC plan as the unique equilibrium.

In the general framework presented above, whether LTC is sufficient to sustain FC outcomes depends on whether finite sequences of policy instruments can uniquely pin down allocations. Our main result, which we prove in the following section, is that an equivalence result between FC and LTC outcomes (with sufficiently large but finite L) holds in a general class of models for which an inherited τ_t^L, together with a commitment to τ_{t+L}, uniquely pin down all the problematic variables appearing in the constraints at t. This implies that FC outcomes can be sustained with a significantly lighter requirement on the policymaker's ability to control future policies than the one assumed in the standard Ramsey-optimal policy literature.

3 Equivalence result

In this section we first introduce some new notation that will be convenient for deriving our main result. We then state our key assumption on the mapping from sequences of policies to sequences of allocations and finally prove the equivalence result between the FC and LTC equilibria under this assumption. We apply standard proofs of recursivity from Stokey and Lucas (1989) to our setup. Hence, readers that are more interested in the applications can skip much of this section if desired, and simply familiarize themselves with **Assumption 3***. This is the key assumption that must be satisfied for our equivalence result to hold, and in Section 4 we show that this assumption is satisfied in several optimal fiscal policy models. All relegated proofs from this section can be found in Appendix A.

3.1 Competitive equilibrium

It is convenient at this point to summarize the variables of the model using $y_t \equiv (b_t, g_t, c_t, p_t, \tau_t) \in Y \subset B \times G \times C \times P \times T$. A sequence $\mathbf{y} \equiv \{y_t\}_{t=0}^{\infty} \in Y^{\infty}$ is denoted a plan. We denote a subsequence starting from time t by $\mathbf{y}_t \equiv \{y_s\}_{s=t}^{\infty} \in Y^{\infty}$. The definition of competitive equilibrium, **Definition 1**, can be straightforwardly restated as a plan satisfying (2), (3), and (4).

Definition 5. *The constraints (2), (3), and (4) define a time-invariant correspondence* $\Gamma^* : Y \mapsto Y^N$*. Let* $Y \subset B \times G \times C \times P \times T$ *be such that* $\Gamma^*(y)$ *is non-empty for all* $y \in Y$*.*

A plan $\mathbf{y}$ thus satisfies competitive equilibrium if and only if $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$ for all $t = 0, 1, ...$ The presence of any terms besides b_{t+1} in determining the feasibility of y_t derives from the problematic elements in (4). We have defined Y as a set such that for any $y_t \in Y$, there exists a continuation sequence starting from $t + 1$ which satisfies competitive equilibrium. Note that given this definition, non-emptiness of the correspondence Γ^* simply requires the existence of at least one competitive equilibrium plan.

At $t = 0$ the only predetermined variables are $(b_0, g_0) \in B \times G$. The set of competitive equilibrium plans from a given initial state (b_0, g_0) is denoted by $\Pi^*(b_0, g_0)$, which is constructed as follows. For

any $(b_0, g_0) \in B \times G$:

$$\Pi^*(b_0, g_0) = \{\{y_t\}_{t=0}^{\infty} : c_0 \in C, p_0 \in P, \tau_0 \in T, y_0 = (b_0, g_0, c_0, p_0, \tau_0), (y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t) \ \ \forall t = 0, 1, ...\} \tag{14}$$

Define $B^* \in B \times G$ as the set of initial conditions for which at least one competitive equilibrium plan exists. $\Pi^*(b_0, g_0)$ is thus non-empty for all $(b_0, g_0) \in B^*$. From now on we take as a maintained assumption that B^* is non-empty, and restrict ourselves to initial conditions in B^*. It is also worth noting at this point that since we can always construct a competitive equilibrium plan by truncating an existing plan at time t, any time-t pair (b_t, g_t) is on a competitive equilibrium plan if and only if it is in B^*. Additionally, any subsequence $\mathbf{y}_t$ satisfies all competitive equilibrium constraints dated t and onwards if and only if $\mathbf{y}_t \in \Pi^*(b_t, g_t)$. The representative agent's utility from $t = 0$ for a given plan, $\mathbf{y}$, is given by

$$u^*(\mathbf{y}) = \sum_{t=0}^{\infty} \beta^t r(c_t, b_t, g_t, \tau_t) \tag{15}$$

where it is understood that the values of (c_t, b_t, g_t, τ_t) used to evaluate r are taken as the relevant elements of y_t.

3.2 Full Commitment

The FC government chooses and commits to an entire path $\{\tau_t\}_{t=0}^{\infty} \in T^{\infty}$ at $t = 0$, taking the initial state $(b_0, g_0) \in B^*$ as given. It is now convenient to restate our Assumptions 1 and 2 with our new notation.

Assumption 1*. *Given any $(b_0, g_0) \in B^*$, for all $\mathbf{y} \in \Pi^*(b_0, g_0)$, $\lim_{n\to\infty} \sum_{t=0}^{n} \beta^t r(c_t, b_t, g_t, \tau_t)$ exists, although it may be plus or minus infinity.*

Assumption 2*. *Given any $(b_0, g_0) \in B^*$, any path for government policy, $\{\tau_t\}_{t=0}^{\infty} \in T^{\infty}$ which lies on a competitive equilibrium plan $\mathbf{y} \in \Pi^*(b_0, g_0)$ lies on no other plans contained in $\Pi^*(b_0, g_0)$.*

The first assumption ensures that discounted utility converges to a limit, and is hence defined. The second is an invertibility assumption, ensuring that the government is able to pin down a unique path for competitive equilibrium given a path for tax rates. Under this assumption we can equivalently define the government's problem as choosing a path for taxes, or simply choosing the

associated plan, $\mathbf{y}$. This allows us to state the FC government's problem, for any $(b_0, g_0) \in B^*$, as

$$V^*(b_0, g_0) = \sup_{\mathbf{y} \in \Pi^*(b_0, g_0)} u^*(\mathbf{y}) \tag{FC}$$

where $V^* : B^* \mapsto \bar{\mathbb{R}}$ is the supremum, giving the value on an optimal plan. The first two assumptions ensure that this exists and is well defined for any $(b_0, g_0) \in B^*$, since there is always at least one competitive equilibrium plan, and all plans lead to well-defined discounted utilities. For any $(b_0, g_0) \in B^*$, denote the set of plans which achieve the supremum by $\mathbf{y}^{FC}(b_0, g_0)$.

3.3 Limited-Time Commitment

The aim of this subsection is to introduce sufficient conditions such that the solutions to the FC and LTC problems coincide, allowing us to restate the LTC problem. We first define the state vector for the LTC game when the governments have L periods of commitment: $x_t \equiv (b_t, g_t, \tau_t, ..., \tau_{t+L-1}) \in B \times G \times T^L$. This is the state that the time-t government inherits: the natural states, b_t and g_t, and the pre-committed taxes, τ_t to τ_{t+L-1}.[11] The government then chooses τ_{t+L}. Notice that any sequence of $\mathbf{y}$ defines a sequence of $\mathbf{x}$ using the definition of x_t: the elements in $(y_t, ..., y_{t+L-1})$ give us x_t.

In Section 2.3, we introduced **Assumption 2-LTC** in order to define the government maximization problem with LTC. We now replace it with a stronger assumption, which is our key requirement for equivalence of LTC and FC outcomes.

Assumption 3*. *There exists an L, with $N \leq L < \infty$, such that for any $t = 0, 1, ...$, any $(\hat{b}_t, \hat{g}_t) \in B^*$ and implied $\hat{g}_{t+1} = \Gamma^g(\hat{g}_t)$, and any sequence $\{\hat{\tau}_s\}_{s=t}^{t+L} \in T^{L+1}$:*

1. *All competitive equilibrium plans $\mathbf{y}_t \in \Pi^*\left(\hat{b}_t, \hat{g}_t\right)$ for which $\{\tau_s\}_{s=t}^{t+L} = \{\hat{\tau}_s\}_{s=t}^{t+L}$ have the same values of y_t, and hence b_{t+1} from (2), which we denote $\hat{y}_t$ and $\hat{b}_{t+1}$.*
2. *All competitive equilibrium plans $\mathbf{y}_{t+1} \in \Pi^*\left(\hat{b}_{t+1}, \hat{g}_{t+1}\right)$ for which $\{\tau_s\}_{s=t+1}^{t+L} = \{\hat{\tau}_s\}_{s=t+1}^{t+L}$ have the same values of any elements of $\{b_s, c_s, p_s\}_{s=t+1}^{t+N}$ which are problematic from the perspective of time t.*

[11] We are now omitting the (redundant) constant state 1 that allowed us to nest the NC equilibrium in Section 2. This is because we only focus on non-zero degree of commitment in this section.

This assumption is crucial in overcoming the time-inconsistency problems inherent in this framework, since it guarantees that L periods of commitment restrict the admissible competitive equilibrium allocations sufficiently. In particular, point 1 defines a function that maps $(b_t, g_t, \tau_t, ..., \tau_{t+L})$ into a single value of (y_t, b_{t+1}), which is the only value consistent with competitive equilibrium conditions. In other words, it requires that we can uniquely pin down all variables at time t once we add the government's choice, τ_{t+L}, to the state, x_t. This allows us to evaluate time t utility and compute the following value of the endogenous state, b_{t+1}. Note that this assumption does not imply that time t utility, $r(c_t, b_t, g_t, \tau_t)$, becomes predetermined from the point of view of time t's government. It can still be affected via the endogenous response of c_t to the government's choice of τ_{t+L}. Since the functions defining competitive equilibrium conditions, given in (2), (3), and (4), do not explicitly depend on time, this mapping is time invariant.

In point 2, the assumption defines a mapping from $(b_{t+1}, g_{t+1}, \tau_{t+1}, ..., \tau_{t+L})$ to the problematic elements of $\{b_{t+s}, c_{t+s}, p_{t+s}\}_{s=1}^{N}$. This restricts the ability of the time-$t+1$ government to alter the problematic variables which appear in the time-t government's constraints. This is because all competitive equilibria which are reachable given her state x_{t+1} feature the same values of these variables.

Note that by point 1 of **Assumption 3*** and given x_t, choosing τ_{t+L} pins down a unique x_{t+1} (and vice versa), hence we can equivalently state the government's problem as one of choosing x_{t+1} given x_t. The time-$t+1$ government then inherits the state x_{t+1} and chooses x_{t+2}, and so on. It is worth stressing that this assumption does not imply that each government in the LTC game has only one feasible choice. In all of the examples we present below, the time-t government has many feasible choices of τ_{t+L}, and will *optimally* choose the value consistent with the FC plan.

We next need to establish the time-t government's choices for x_{t+1} which belong to a competitive equilibrium path given the state x_t, which we do in the following lemma. First define the restricted set:

$$X = \left\{x \in B \times G \times T^L : x \text{ lies on at least one path } \mathbf{y} \in \Pi^*(b_0, g_0) \text{ for some } (b_0, g_0) \in B^*\right\} \tag{16}$$

This is the set of values for the state x_t which are compatible with competitive equilibrium.[12]

[12]In Appendix C we provide an algorithm for computing this set recursively, which we apply in our numerical work.

Lemma 1. *There exists a time-invariant transition function $\Gamma : X \mapsto X$ defined s.t. x_{t+1} is on t least one competitive equilibrium path given x_t iff $x_{t+1} \in \Gamma(x_t)$. For all $x_t \in X$, $\Gamma(x_t)$ is non-empty.*

We relegate the proof to Appendix A, and here simply note that the key feature of this lemma is that it has converted all of the constraints facing the time-t government, some of which were of the form in (4) which contained problematic variables, into a standard constraint, compatible with the usual dynamic programming methods. Specifically, the feasibility of the time-t choice of x_{t+1} does not depend on any choices made by future governments, for example x_{t+2}.

Assumption 3* is crucial in proving this result. Since x_{t+1} is restricted to lie in X, we know that there exists at least one competitive equilibrium path starting from $t+1$ for any choice of x_{t+1}. Thus to check whether x_{t+1} is on at least one competitive equilibrium path starting from t we only need to check the time-t constraints: does $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$? **Assumption 3*** implies that we can check if this is true by looking only at x_t and x_{t+1}, since they pin down unique values of all the variables needed to check if $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$ regardless of the values of $x_{t+2}, x_{t+3}, ...$

Finally, let $P = \{(x, y) \in X \times X : y \in \Gamma(x)\}$ be the graph of $\Gamma(x)$, and redefine the utility function as $F : P \mapsto \mathbb{R}$ such that $F(x_t, x_{t+1}) = r(c_t, b_t, g_t, \tau_t)$. Again, note that **Assumption 3*** allows us to back out a unique $y_t = (b_t, g_t, c_t, p_t, \tau_t)$ given (x_t, x_{t+1}), which is what allows this reformulation. We make the following boundedness assumption on the utility function.[13]

Assumption 4*. *The environment is such that $F(x, y)$ is bounded for all $x \in X$ and $y \in \Gamma(x)$.*

Under these assumptions, a Markov perfect equilibrium of the LTC game with L periods of commitment can be re-expressed as

$$v(x_t) = \sup_{x_{t+1} \in \Gamma(x_t)} \{F(x_t, x_{t+1}) + \beta v(x_{t+1})\}, \ \ \forall x_t \in X \tag{LTC}$$

This definition of the LTC game is equivalent to the definition from Section 2.3, which defined MPE as a fixed point of (7), but the statement is simplified by the above results. In particular, we are able to express both the transition, Γ, and the utility, F, in terms only of today's state, x_t, and today's choice, x_{t+1}. This means that, stated this way, no future choice variables enter the constraints,

[13]Note that this assumption does allow unbounded utility functions, such as CRRA over consumption, as long as competitive equilibrium places a bound on the feasible levels of utility.

allowing a standard recursive formulation. In terms of the notation of Section 2.3, our assumptions allow us to remove the dependence of ϕ^{LTC} on the policy function of future governments, $\tilde{\tau}^{LTC}$. This allows the return function, F, and transition correspondence, Γ, to depend only on x_t and x_{t+1}.

3.4 Equivalence of LTC and FC

Having restated the FC and LTC games in terms of our notation, we now turn to demonstrating the equivalence between the two equilibrium outcomes. The main proposition is stated below.

Proposition 1. *Consider an L such that* **Assumptions 1* to 4*** *hold, and fix a $(b_0, g_0) \in B^*$. If, in the LTC game, either*

1. *$\{\tau_t\}_{t=0}^{L-1}$ is restricted to be optimal values from the FC game, or*
2. *the time-0 government, in addition to choosing τ_L, is also allowed to choose $\{\tau_t\}_{t=0}^{L-1}$*

then all equilibria of the LTC game generate paths $\mathbf{y} \in \mathbf{y}^{FC}(b_0, g_0)$, and the supremum of time-0 value is $V^*(b_0, g_0)$.

The bulk of the proof rests on establishing the equivalence between the recursive LTC game and the FC problem, which is done in two steps. We first re-express the FC problem as a "Modified Problem" (MP) where the government chooses paths for x_t instead of y_t. We then show that MP has a recursive formulation equivalent to the LTC game.

To set up the Modified Problem, we first need to define plans in terms of our new state variable: $\mathbf{x} \equiv \{x_t\}_{t=0}^{\infty} \in X^{\infty}$. This allows us to define the set of competitive equilibrium plans, $\mathbf{x}$, starting from a given $x_0 \in X$:

$$\Pi(x_0) = \{\{x_t\}_{t=0}^{\infty} \in X^{\infty} : x_{t+1} \in \Gamma(x_t), \;\; t = 0, 1, ...\}$$

This allows us to redefine the path utilities using $u : \Pi(x_0) \mapsto \bar{\mathbb{R}}$ by $u(\mathbf{x}) = \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1})$. We can then define MP as:

$$V(x_0) = \sup_{\mathbf{x} \in \Pi(x_0)} u(\mathbf{x}) \tag{MP}$$

This problem is to maximize utility given an initial state x_0, by choosing a plan $\mathbf{x}$.[14] Notice that there are thus two differences from the original FC problem. Firstly, the state for the FC problem

[14] **Lemma A1** in Appendix A establishes that the supremum in MP is well defined.

is just (b_0, g_0), but the state here is $x_0 = (b_0, g_0, \tau_0, ..., \tau_{L-1})$, so the MP problem maximizes utility subject to the initial taxes being taken as given. Secondly, the MP chooses plans for $\mathbf{x}$, whereas the FC problem chooses plans for $\mathbf{y}$. However, the following lemma demonstrates that under our assumptions, choosing plans for $\mathbf{x}$ or $\mathbf{y}$ is equivalent.

Lemma 2. *For all $(b_0, g_0) \in B^*$, each $\mathbf{y} \in \Pi^*(b_0, g_0)$ implies a unique $\mathbf{x} \in \Pi(x_0)$ for some $x_0 \in X$. Conversely, for all $x_0 \in X$, each $\mathbf{x} \in \Pi(x_0)$ implies a unique $\mathbf{y} \in \Pi^*(b_0, g_0)$ for some $(b_0, g_0) \in B^*$.*

Proof in Appendix A. For any $(b_0, g_0) \in B^*$, define the set $Q(b_0, g_0)$ as follows:

$$Q(b_0, g_0) \equiv \left\{(\tau_0, ..., \tau_{L-1}) \in T^L : (b_0, g_0, \tau_0, ..., \tau_{L-1}) \in X\right\}$$

This is the set of L initial government choices that lie on competitive equilibrium paths for a given initial state. The following lemma establishes that as long as these initial choices are chosen correctly, the same paths solve MP and FC, and lead to the same maximized value.

Lemma 3. (FC = MP) *For any $(b_0, g_0) \in B^*$,*

1. *$V((b_0, g_0, \tau_0, ..., \tau_{L-1})) = V^*(b_0, g_0)$ for any $(\tau_0, ..., \tau_{L-1})$ from an optimal FC plan in $\mathbf{y}^{FC}(b_0, g_0)$*
2. *$\sup_{(\tau_0,...,\tau_{L-1}) \in Q(b_0,g_0)} V((b_0, g_0, \tau_0, ..., \tau_{L-1})) = V^*(b_0, g_0)$*

and the implied optimal plans from MP, $\mathbf{x}$, all generate plans $\mathbf{y}$ which lie in $\mathbf{y}^{FC}(b_0, g_0)$.

Proof in Appendix A. This lemma states that the Modified Problem generates the same maximized utility and paths as the Full Commitment solution, as long as the initial policies are either 1) arbitrarily chosen to be the FC-optimal policies, or 2) chosen to maximize the value attained in MP. Having established the equivalence between the FC and MP problems, all that remains is to establish the equivalence between the MP problem and the LTC game. This is done in the final lemma.

Lemma 4. (MP = LTC) *The function V is the unique solution to (LTC). Additionally, for any $x_0 \in X$, any plan $\mathbf{x} \in \Pi(x_0)$ which satisfies $V(x_t^*) = F(x_t^*, x_{t+1}^*) + \beta V(x_{t+1}^*)$ for $t = 0, 1, ...$ attains the supremum in (MP) for initial state x_0.*

Proof in Appendix A. Intuitively, the MP was constructed to be recursive with x_t as a state. This is precisely the state variable of the LTC game, and given the identical underlying environments, the LTC provides the recursive form of MP. The boundedness condition in **Assumption 4*** then ensures that the MP value, V, provides the unique solution to LTC.

Combining the two equivalences between FC and MP, and MP and LTC delivers the proof of **Proposition 1**.

Proof of Proposition 1. In **Lemma 4** we showed that the unique equilibrium of the LTC game is the solution to MP, for an arbitrary x_0. To prove point 1, note that if $(\tau_0, ..., \tau_{L-1})$ are chosen to be optimal values from the FC problem then point 1 of **Lemma 3** implies the equivalence of MP and FC, thus establishing the equivalence of LTC and FC. For point 2, where the time-0 government is also allowed to choose $(\tau_0, ..., \tau_{L-1})$, note that we can then split the time-0 problem of choosing $(\tau_0, ..., \tau_L)$ into two steps: 1) Choose τ_L given $(\tau_0, ..., \tau_{L-1})$, which we know from **Lemma 4** solves MP for the implied $x_0 = (b_0, g_0, \tau_0, ..., \tau_{L-1})$. This yeilds maximised value $V((b_0, g_0, \tau_0, ..., \tau_{L-1}))$. 2) Choose $(\tau_0, ..., \tau_{L-1})$ to maximize $V((b_0, g_0, \tau_0, ..., \tau_{L-1}))$. In this case the time-0 problem can be expressed as $\sup_{(\tau_0,...,\tau_{L-1})\in Q(b_0,g_0)} V((b_0, g_0, \tau_0, ..., \tau_{L-1}))$, which we know from point 2 of **Lemma 3** to be equal to the FC value $V^*(b_0, g_0)$. □

This completes the proof of **Proposition 1**. We have thus proved that the solution to the FC problem can be supported as the unique Markov perfect equilibrium of the LTC game, as long as the initial policies, $\{\tau_t\}_{t=0}^{L-1}$, are either arbitrarily chosen to be the optimal FC choices, or if the time-0 government is also allowed to optimally choose these policies.

4 Equivalence in specific models of fiscal policy

In this section we show how our equivalence result applies in several models of optimal fiscal policy that have been studied in the literature. We start from bond-only economies such as Lucas and Stokey (1983) and Faraglia et al. (2014) and then move on to study optimal capital taxation in a version of the Chamley (1986) and Judd (1985) economies. In all these models, we illustrate that **Assumption 3*** holds and hence FC outcomes can be supported as symmetric Markov equilibria

in the LTC game.[15]

We restrict the set of exogenous states to just government spending for simplicity, and it is worth noting that all our results go through if extra exogenous states (such as deterministically time-varying productivity) satisfying the Markov property are added. We restrict exposition to two popular models in this section, and discuss additional models where our theorem holds in Appendix B.

4.1 Economies without capital

We start our analysis by considering a deterministic economy without capital and generalizing the example presented in Section 2.4, by allowing for time-varying government spending and long-maturity bonds. An exogenous stream of expenditure, $\{\mathrm{g}_t\}_{t=0}^{\infty}$, which for simplicity we assume to be purely wasteful, needs to be financed with labor income taxes, $\left\{\tau_t^l\right\}_{t=0}^{\infty}$, and debt, $\{\mathrm{b}_{t+N}\}_{t=0}^{\infty}$, with generic maturity $N \in [1, \infty)$. This model encompasses a deterministic version of the model studied by Lucas and Stokey (1983) as well as models of optimal debt maturity as in Faraglia et al. (2014).[16] The choice of the label N for maturity is not accidental, as this will indeed turn out to coincide with the definition of N from our general formulation: variables N periods ahead appear in the constraints.

A representative household has preferences defined over sequences of private consumption $\{\mathrm{c}_t\}_{t=0}^{\infty}$ and labor effort $\{\mathrm{l}_t\}_{t=0}^{\infty}$:

$$\sum_{t=0}^{\infty} \beta^t u(\mathrm{c}_t, \mathrm{l}_t) \tag{17}$$

with standard assumptions $u_{\mathrm{c}} > 0$, $u_{\mathrm{cc}} < 0$, $u_{\mathrm{l}} < 0$, $u_{\mathrm{ll}} < 0$. Her budget constraint is given by

$$\mathrm{c}_t + \mathrm{q}_t \mathrm{b}_{t+N} = \mathrm{w}_t \mathrm{l}_t \left(1 - \tau_t^l\right) + \mathrm{b}_t \tag{18}$$

where q_t is the price of a bond issued at t that repays one unit of consumption at $t + N$.[17] Output

[15] We relegate the discussion of the restrictions on the environment necessary for the boundedness condition in **Assumption 4*** to hold to Appendix B. **Assumption 1*** holds in all of our settings, and **Assumption 2*** is implied by **Assumption 3***.

[16] The analysis can easily be extended to multiple maturities, but we restrict ourselves to one bond for expositional simplicity.

[17] In the case of $N > 1$ we are considering a long-bond economy with "no buy-back": governments cannot repurchase bonds before maturity. This is for expositional purposes, and our results also apply in the case of buy-back.

equals labor effort, hence the resource constraint reads

$$\mathrm{c}_t + \mathrm{g}_t = \mathrm{l}_t \tag{19}$$

and firms' profit maximization implies a unit wage: $\mathrm{w}_t = 1$. The government's budget is implicitly defined by the agent's budget constraint and the resource constraint.

The household's first order conditions with respect to consumption, labor effort and bonds, together with the resource constraint, can be summarized by an intratemporal optimality condition and a Euler equation:

$$-\frac{u_{\mathrm{l}}(\mathrm{c}_t, \mathrm{c}_t + \mathrm{g}_t)}{u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{c}_t + \mathrm{g}_t)} = 1 - \tau_t^l \tag{20}$$

$$\mathrm{q}_t u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{c}_t + \mathrm{g}_t) = \beta u_{\mathrm{c}}(\mathrm{c}_{t+N}, \mathrm{c}_{t+N} + \mathrm{g}_{t+N}) \tag{21}$$

This completes the description of the model, allowing us to map it into the general framework of Section 3. In the general notation we have $b_t = (\mathrm{b}_t, ..., \mathrm{b}_{t+N-1})$, $g_t = \mathrm{g}_t$, $c_t = (\mathrm{c}_t, \mathrm{l}_t)$, $p_t = \mathrm{q}_t$, and $\tau_t = \tau_t^l$. Note that we have already imposed bond market clearing implicitly and solved out for the real wage. The transition Γ^* is defined by the equations (18), (19), (20), and (21), and an assumed transition Γ^g for g_t. Note that, according to our definition, c_{t+N} is thus the only problematic variable in the time-t constraints, since it appears in (21). We refer to the previous literature for a derivation of FC optimal policy in this model. Here we limit ourselves to a brief discussion of the difference between FC and NC equilibria in this context.

As in the example in Section 2.4, the Euler equation (21) highlights the source of time-inconsistency of the FC policy in this model. At $t = 0$, the FC government has an incentive to use the initial allocation to decrease the value of outstanding initial debt b_0, and hence reduce the distortions required to finance expenditure.

The properties of the solution under NC depend crucially on the assumptions about government spending. For $N = 1$, and if government spending is assumed exogenous and constant, Krusell et al. (2004) prove the existence of a "step function" equilibrium which supports multiple steady states for government debt.[18] Debortoli and Nunes (2013) study a version of this economy with $N = 1$ which also features endogenous government spending valued in utility. As discussed above,

[18] In principle, it is also possible for a smooth equilibrium to exist, although it is challenging to find numerically, making its characterization hard.

the FC equilibria features a long run level of debt which depends on the initial level of debt: initial conditions matter. The NC government, on the other hand, has a debt policy which converges to a steady state level b^* regardless of initial conditions. Overall, there are thus large differences in debt dynamics between the solutions to these models under FC and NC.

In the LTC game with an arbitrary L periods of commitment the government inherits the following states at time t. Firstly there are the natural states, g_t and $(b_t, ..., b_{t+N-1})$. Then there are the pre-committed taxes, $(\tau_t^l, ..., \tau_{t+L-1}^l)$. The government then chooses τ_{t+L}^l. Note the similarity to the NC problem, where the government only inherits the natural states, and chooses τ_t^l.

We now show that the FC equilibrium can be supported in the LTC game with $L = N$ periods of commitment. In the Lucas and Stokey (1983) economy, which has $N = 1$, this means that just one period of commitment is sufficient to recover FC. To prove this, we need to show that our key assumption on the mapping between sequences of taxes and sequences of allocations, **Assumption 3***, holds in this model. In other words, we need to show that given the natural states $(g_t, b_t, ..., b_{t+N-1})$, if we fix $(\tau_t^l, ..., \tau_{t+N}^l)$, then (i) we pin down all of $y_t = (b_t, ..., b_{t+N-1}, g_t, c_t, l_t, q_t, \tau_t^l)$, of which we only need to check (c_t, l_t, q_t), and b_{t+N}, and (ii) the problematic variable c_{t+N} is fixed given $(g_{t+1}, b_{t+1}, ..., b_{t+N}, \tau_{t+1}^l, ..., \tau_{t+N}^l)$. To see that this is the case, consider equation (20). Given a tax rate τ_t^l (and an exogenous g_t), this is one equation pinning down one unknown, namely c_t.[19] Hours l_t can then be easily recovered from the resource constraint. Hence a sequence $\left\{\tau_t^l, ..., \tau_{t+N}^l\right\}$ pins down a sequence of consumption and hours $\{(c_{t+j}, l_{t+j})\}_{j=0}^{t+N}$. The bond price, q_t can then be recovered from (21) and bonds b_{t+N} from the budget constraint (18), given an outstanding level of debt b_t.

Having shown that our key assumption holds, we have proved that the FC solution can be supported by the LTC game. An important result of this analysis is that the degree of commitment necessary to achieve FC outcomes depends crucially on the the maturity of debt. The longer this maturity, the higher the number of periods of commitment required. Hence for a given planning horizon for fiscal policy, economies with longer debt maturity appear to be more prone to the welfare costs of imperfect commitment.

[19] Since this equation is nonlinear, an additional, weak regularity assumption is required in order to ensure that the solution is unique, so that a given tax rate pins down a unique allocation. This amounts to ensuring that the left hand side of (20) is either strictly increasing or decreasing in c_t for $c_t \geq 0$, a condition that is satisfied for standard utility functions, such as separable isoelastic utility in c_t and l_t.

4.2 The role of initial conditions

The equivalence between LTC and FC outcomes proved in Section 3 relies on initial policy instruments being consistent with the FC plan. We now discuss the consequences of letting governments with LTC inherit arbitrary initial policies. By **Lemma 4**, we know that for any model, the LTC outcome coincides with the outcome of a FC Ramsey plan restricted to start from the same arbitrary initial policy. Hence even if the LTC game is initialized with "incorrect" initial policies, the remaining policies will be chosen optimally in the sense that a FC Ramsey planner restricted to the same initial policies would choose the same sequence. However, for specific models, we are able to provide a more detailed characterization of the equilibrium. In particular, here we focus again on the model without capital.

We simplify the exposition as in the example in Section 2.4 by restricting the analysis to one-period bonds (N=1) and constant government spending.[20] For this economy, we have established that, starting from initial conditions given by the FC policy sequence, LTC sustains FC outcomes with a single period of commitment. We now ask the question of what happens if a government with LTC game inherits an arbitrary initial policy, potentially different from the one implied by the FC policy path. We show that the economy converges in one period to another FC equilibrium, consistent with a different level of initial debt. We also provide simple formulas to evaluate the welfare cost of starting from a "wrong" initial policy.

From the intratemporal optimality condition (20), imposing constant g_t, we can obtain hours as an implicit function of the tax rate: $\mathrm{l}_t = l(\tau_t^l)$. Using this function, the government budget constraint in period 0 can then be expressed as

$$u_c(l(\tau_0^l) - \mathrm{g}, l(\tau_0^l)) \left(\mathrm{b}_0 + \mathrm{g} - \tau_0^l l(\tau_0^l)\right) = \beta u_c(\mathrm{c}_1, \mathrm{l}_1)\mathrm{b}_1. \tag{22}$$

Let $\mathrm{a}_t \equiv u_c \left(l\left(\tau_t^l\right) - \mathrm{g}, l\left(\tau_t^l\right)\right)\left(\mathrm{b}_t + \mathrm{g} - \tau_t^l l(\tau_t^l)\right)$ and note that this variable is a function only of the inherited level of debt and the current tax rate: $\mathrm{a}_t = a\left(\mathrm{b}_t, \tau_t^l\right)$. The economic interpretation of this variable is the (marginal utility) value of the resources that the time-t government needs to raise on the bond market.

[20] This assumption can be easily relaxed without affecting the main insight. However, perfect tax smoothing then arises only under a stronger assumption on preferences, such as separable, CRRA in both consumption and labor.

Let $s\left(\tau_t^l\right) \equiv u_c\left(l\left(\tau_t^l\right)-\mathrm{g}, l\left(\tau_t^l\right)\right)\left(\mathrm{g}-\tau_1^l l\left(\tau_t^l\right)\right)$. Adding and subtracting $s\left(\tau_1^l\right)$ on the right-hand side of (22) yields

$$\mathrm{a}_0=\beta\left(\mathrm{a}_1+s\left(\tau_1^l\right)\right). \tag{23}$$

Note that that the problem of the government at $t=0$ is affected by $\left(\mathrm{b}_0, \tau_0^l\right)$ only through their effect on a_0. This is because this government cannot affect hours worked at $t=0$. Hence the government's optimization problem can be formulated in the following recursive form.[21]

$$W(\mathrm{a})=\max_{\left(\mathrm{a}', \tau'\right)} \beta\left[u\left(l\left(\tau'\right)-\mathrm{g}, l\left(\tau'\right)\right)+W\left(\mathrm{a}'\right)\right] \tag{24}$$

subject to the transition $\mathrm{a}'=\beta^{-1} \mathrm{a}-s\left(\tau'\right)$. Note that this recursive form ignores contemporaneous utility, which is an any case fixed from the government's point of view. The FC policy for this model is fully characterized by two functions: $\tau_{0,FC}^l(\mathrm{b}_0)$ for $t=0$ and $\tau_{FC}^l(\mathrm{b}_0)$ with $\tau_t^l=\tau_{FC}^l(\mathrm{b}_0)$ for all $t \geq 1$. The government chooses a perfectly smooth tax from $t=1$ onwards and uses the tax rate at $t=0$ to affect the utility value of initial debt by distort the initial allocation in order to decrease the amount of tax distortions needed to finance expenditure and service the debt. The allocation is also constant from $t=1$ onwards.

In order for debt not to explode with a constant tax rate and constant hours and consumption, it has to be the case that a_t is also constant. Hence, we can find the optimal tax rate from time 1 onwards from the transition equation for a in steady-state: $\tau^*(\mathrm{a}_0)=s^{-1}\left(\frac{1-\beta}{\beta} \mathrm{a}_0\right)$ and the value function satisfies

$$W(\mathrm{a}_0)=\frac{\beta}{1-\beta}\left[u\left(l\left(\tau^*\left(\mathrm{a}_0\right)\right)-\mathrm{g}, l\left(\tau^*\left(\mathrm{a}_0\right)\right)\right)\right]. \tag{25}$$

Total welfare starting from arbitrary initial conditions (b_0, τ_0^l) is then given by

$$V\left(\mathrm{b}_0, \tau_0^l\right)=u\left(l\left(\tau_0^l\right)-\mathrm{g}, l\left(\tau_0^l\right)\right)+W(\mathrm{a}_0). \tag{26}$$

We now argue that the LTC policy and allocation starting from $\left(\mathrm{b}_0, \tau_0^l\right)$, with $\tau_0^l \neq \tau_{0,FC}^l\left(\mathrm{b}_0\right)$ converges to another FC policy (and allocation), indexed by a different debt level.

[21] This is an alternative formulation relative to the more general recursive formulation used to prove **Proposition 1**. It holds in this model because the welfare-relevant component of the allocation $(\mathrm{c}_t, \mathrm{l}_t)$ is fixed from the point of view of the government dated t.

Let $\tilde{\mathrm{b}}_0$ be the solution to the non-linear equation $a\left(\tilde{\mathrm{b}}_0, \tau^l_{0,FC}\left(\tilde{\mathrm{b}}_0\right)\right) = a\left(\mathrm{b}_0, \tau^l_0\right)$.[22] Then, the government at time 0 solves the problem defined in (24) starting from $\mathrm{a}_0 = a\left(\tilde{\mathrm{b}}_0, \tau^l_{0,FC}\left(\tilde{\mathrm{b}}_0\right)\right)$ and the policy and allocation from $t = 1$ onwards will coincide with the ones implied by the FC equilibrium starting from $\tilde{\mathrm{b}}_0$. In particular, we will have $\tau^l_t = \tau^*\left(\mathrm{a}_0\right) = \tau^l_{FC}\left(\tilde{\mathrm{b}}_0\right)$ for all $t \geq 1$. In order to assess the welfare cost of starting from any initial tax, it is sufficient to compare the value attained by the FC policy starting from $t = 0$ with the value defined in (26).

In Appendix B we consider a model with capital and prove a similar result on the role of initial policy conditions, under the assumption of linear utility from consumption.

4.3 Capital and labor taxes

We now consider optimal fiscal policy in economies with capital, as in Chamley (1986) and Judd (1985). In particular, we consider a model that nests the economies analyzed by Klein et al. (2008) and Debortoli and Nunes (2010). We allow for government consumption to be valued by households and chosen by the government, making the policy instruments labor taxes, τ^l_t, capital taxes, τ^k_t, and government spending, g_t. We also let the capital utilization rate, v_t, be endogenous.

Crucially, we assume that the government must balance its budget period by period. The importance of balanced-budget rules was first studied by Stockman (2001) under the assumption of FC. While Stockman (2001) allows for an arbitrary constant level of debt, we set this level to zero and explore the role of balanced-budget rules for the possibility of sustaining FC outcomes under LTC. We defer discussion of more general budget rules and time-to-build to Section 4.4.

We follow Debortoli and Nunes (2010) in the model description.[23] Household preferences are represented by the utility function

$$\sum_{t=0}^{\infty} \beta^t u(\mathrm{c}_t, \mathrm{g}_t, \mathrm{l}_t) \tag{27}$$

with $u_{\mathrm{c}} > 0$, $u_{\mathrm{cc}} < 0$, $u_{\mathrm{g}} > 0$, $u_{\mathrm{gg}} < 0$, $u_{\mathrm{l}} < 0$, $u_{\mathrm{ll}} < 0$. Output is produced using a Cobb-Douglas technology that combines capital (with an endogenous utilization rate v_t) and labor:

$$\mathrm{y}_t = \left(\mathrm{v}_t \mathrm{k}_t\right)^{\alpha} \mathrm{l}_t^{1-\alpha}. \tag{28}$$

[22] Under standard parametrizations of the utility function there is a unique solution.

[23] Klein et al. (2008)'s models can be recovered by removing endogenous capital utilization. Klein and Ríos-Rull (2003) add stochastic shocks to this framework, while we focus on its deterministic version.

Following Greenwood et al. (2000), capital depreciates at rate $\delta(\mathrm{v}_t)$ with $\delta_{\mathrm{v}} > 0$ and $\delta_{\mathrm{vv}} > 0$. Depreciation is increasing in the rate of utilization, giving a well defined trade-off which determines the optimal level of utilization. Endogenous utilization makes the capital taxation problem more tractable, because even at time 0 the government faces the cost that higher capital taxes will lower utilization. The resource constraint of the economy reads:

$$\mathrm{c}_t + \mathrm{k}_{t+1} - (1 - \delta(\mathrm{v}_t))\,\mathrm{k}_t + \mathrm{g}_t = (\mathrm{v}_t \mathrm{k}_t)^{\alpha}\, \mathrm{l}_t^{1-\alpha}. \tag{29}$$

Households consume, supply labor, invest in capital and choose its utilization rate, renting utilized capital to firms. Combining the households' and firms' optimality conditions leads to the following conditions.[24]

$$-\frac{u_{\mathrm{l}}(\mathrm{c}_t, \mathrm{g}_t, \mathrm{l}_t)}{u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{g}_t, \mathrm{l}_t)} = \left(1 - \tau_t^l\right)(1-\alpha)\,(\mathrm{v}_t \mathrm{k}_t)^{\alpha}\, \mathrm{l}_t^{-\alpha} \tag{30}$$

$$\left(1 - \tau_t^k\right) \alpha \mathrm{v}_t^{\alpha} \mathrm{k}_t^{\alpha-1} \mathrm{l}_t^{1-\alpha} = \delta'(\mathrm{v}_t) \tag{31}$$

$$u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{g}_t, \mathrm{l}_t) = \beta u_{\mathrm{c}}(\mathrm{c}_{t+1}, \mathrm{g}_{t+1}, \mathrm{l}_{t+1}) \left[\alpha\,(\mathrm{v}_{t+1}\mathrm{k}_{t+1})^{\alpha-1}\, \mathrm{l}_{t+1}^{1-\alpha} \left(1 - \tau_{t+1}^k\right) + 1 - \delta(\mathrm{v}_{t+1})\right] \tag{32}$$

We assume that the government budget constraint has to be balanced in every period. This imposes the restriction that $\tau_t^l \mathrm{w}_t \mathrm{l}_t + \tau_t^k \mathrm{r}_t \mathrm{v}_t \mathrm{k}_t = \mathrm{g}_t$, where r_t is the rental rate of capital. Combined with the firms' first order conditions for capital and labor, this gives the condition

$$\left[\alpha \tau_t^k + (1-\alpha)\,\tau_t^l\right] (\mathrm{v}_t \mathrm{k}_t)^{\alpha}\, \mathrm{l}_t^{1-\alpha} = \mathrm{g}_t. \tag{33}$$

The households' budget constraint is implied by the government's budget and the resource constraint. That the government must balance its budget every period turns out to be an important assumption in order to obtain equivalence of LTC and FC in this economy, as will be made clear. This completes the statement of the model. In the general notation of Section 3 we have $b_t = \mathrm{k}_t$, $c_t = (\mathrm{c}_t, \mathrm{l}_t, \mathrm{v}_t)$, and $\tau_t = (\tau_t^k, \tau_t^l, \mathrm{g}_t)$. There are no exogenous states (g_t) and we have solved out for all prices (p_t). The transition correspondence, Γ^*, is defined by (29), (30), (31), (32), (33).

A source of time-inconsistency in this model is the incentive of the government to promise low capital taxes in order to foster capital accumulation and then to tax capital ex post once it has been

[24] The households' optimality conditions are a standard labor supply condition, utilization choice, and Euler equation for capital. The firms' optimality conditions are the standard labor and capital rental conditions $\mathrm{w}_t = (1-\alpha)\,(\mathrm{v}_t \mathrm{k}_t)^{\alpha}\, \mathrm{l}_t^{-\alpha}$ and $\mathrm{r}_t = \alpha\,(\mathrm{v}_t \mathrm{k}_t)^{\alpha-1}\, \mathrm{l}_t^{1-\alpha}$.

installed. More formally, the government is constrained by the Euler equation, (32), which contains the problematic variables c_{t+1}, v_{t+1} and l_{t+1}, and next period's capital tax, τ^k_{t+1} and government spending g_{t+1}. In the notation of our general formulation we have $N = 1$, with variables one period ahead appearing in the constraints.

Once again we refer to the previous literature for the derivation of the optimal policy under FC and NC. We limit ourselves to proving that FC outcomes can be supported as equilibrium of the LTC game. It will turn out that the government can sustain the FC solution with $L = 1$ periods of commitment in this model. In the LTC game with $L = 1$ periods of commitment the government inherits the following states at time t. Firstly there is the natural state, k_t. Then there are the pre-committed policies, $(\tau^k_t, \tau^l_t, \mathrm{g}_t)$. The government then chooses $(\tau^k_{t+1}, \tau^l_{t+1}, \mathrm{g}_{t+1})$.

We need to show that **Assumption 3*** holds in this model for $L = 1$. In other words, we need to show that given the natural state k_t, if we fix $(\tau^k_t, \tau^l_t, \tau^k_{t+1}, \tau^l_{t+1}, \mathrm{g}_t, \mathrm{g}_{t+1})$, then (i) we pin down all of $y_t = (\mathrm{k}_t, \mathrm{c}_t, \mathrm{l}_t, \mathrm{v}_t, \tau^k_t, \tau^l_t, \mathrm{g}_t)$, of which we only need to check $(\mathrm{c}_t, \mathrm{l}_t, \mathrm{v}_t)$, and k_{t+1}, and (ii) the problematic variables $(\mathrm{c}_{t+1}, \mathrm{l}_{t+1}, \mathrm{v}_{t+1})$ are uniquely pinned down given $(\mathrm{k}_{t+1}, \tau^k_{t+1}, \tau^l_{t+1}, \mathrm{g}_{t+1})$.

To see that this is the case, notice that, given the government's state, $\left(\mathrm{k}_t, \tau^k_t, \tau^l_t, \mathrm{g}_t\right)$, equations (29), (30), (31) and (33) form a system of four (non-linear) equations in four unknowns, namely $(\mathrm{c}_t, \mathrm{l}_t, \mathrm{v}_t, \mathrm{k}_{t+1})$. If this system admits a unique solution, then we satisfy the first requirement. This is the case as long as preferences are such that there are non-separability and/or wealth effects on labor supply. That is, the MRS, $-u_{\mathrm{l}}(\mathrm{c}_t, \mathrm{g}_t, \mathrm{l}_t)/u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{g}_t, \mathrm{l}_t)$, is a function of c_t. This rules out preferences of the Greenwood, Hercowitz and Huffman (1988) form, but admits all preferences such that the MRS is monotone in c_t, given l_t. This allows all standard separable preferences in consumption and leisure for which $u'(c) > 0$ and $u''(c) < 0$ for all c, non-separable preferences of the Cobb-Douglas form, and other non-separable preferences under suitable restrictions.[25] By the same logic, by choosing $\left(\tau^k_{t+1}, \tau^l_{t+1}, \mathrm{g}_{t+1}\right)$, the government thus pins down $(\mathrm{c}_{t+1}, \mathrm{l}_{t+1}, \mathrm{v}_{t+1})$, which are the problematic variables in (32), satisfying the second requirement. Since our main assumption holds, we have shown the equivalence of the LTC and FC solutions in this model. Since, in this model, the Chamley-Judd result of zero capital taxation in the long run holds under FC, we have also shown

[25] We can prove uniqueness recursively. First take the ratio of (31) and (33) to form a single equation in v_t. Since $\delta(\mathrm{v}_t)$ is strictly increasing, this pins down a unique v_t. Next, (33) gives a unique l_t. Given l_t, (30) pins down a unique c_t as long as the MRS is monotone in c_t, as discussed above. Finally, (29) pins down a unique k_{t+1}.

that the Chamley-Judd result can be sustained with a single period of commitment.

It is worth discussing the importance of the balanced budget assumption for this result. The balanced budget equation, (33), is one of the four equations we used to pin down equilibrium with only a finite number of periods of taxes. Intuitively, to sustain commitment, the time-t government must not be able to affect the values of c_t and the other problematic variables, since she disagrees with the time-$t-1$ government about their optimal values.

Pinning down c_t was possible with just one period of commitment in the models without capital of Section 4.1, because in equilibrium consumption was determined by a static goods market clearing condition and labor market optimality condition, summarized by (20). One period of commitment to τ_t^l completely determines c_t in equilibrium, irrespective of the time-t government's choice of τ_{t+1}^l. However, with capital the goods market clearing condition is no longer static, so the time-t government can affect c_t by using $\left(\tau_{t+1}^k, \tau_{t+1}^l, g_{t+1}\right)$ to affect investment.

The balanced budget assumption stops the time-t government from being able to do this, since this would influence government revenue, potentially unbalancing the time-t budget. We showed above that there is a unique c_t which is consistent with balancing the budget if time-t government spending and taxes are predetermined. Thus, the time-t government is unable to deviate from the values of the problematic variables that the time-$t-1$ government intended her to choose when she set the time-t policies, and this inability to deviate is what allows LTC to sustain the FC allocation.

Without assuming a balanced budget our theorem would not hold, and LTC would not support the FC solution. The balanced budget rule effectively plays the role of an extra static equation, replacing the static goods market clearing condition that was lost when capital was added to the model. It is the fact that the rule is static, and not that the budget has to be exactly balanced, which is important. Hence, it is possible to show that constitutionally imposed cyclically-adjusted balanced budget rules would also allow LTC to support the FC outcome.[26]

[26]Relatedly, Halac and Yared (2014) study the role of fiscal rules when present-biased governments cannot commit. We focus on a different source of time-inconsistency, namely forward-looking constraints.

4.4 Additional models

In this section we briefly discuss three additional models where our theorem holds, with detailed expositions and proofs relegated to Appendix B. These are all extensions of the capital and labor taxation model of Section 4.3.

Firstly, we provide an extension where the budget must be balanced across a fixed number of periods. In particular, we suppose that the government can issue one-period bonds, denoted b_t, but that every M periods the government must set $b_{t+1} = 0$ and not issue any bonds. This captures the idea of medium run fiscal constraints placed upon the government, such as yearly balanced-budget restrictions in a quarterly model. In this case LTC can support FC with $L = M$ periods of commitment.

This extension is interesting for two reasons. It illustrates the key role that the length of time over which the budget must be balanced plays in achieving FC in this model. This leads to potentially important policy implications, especially for countries that are implementing multi-annual budget plans, such as several European countries. Moreover, it provides an example where $L > N$, and the number of periods of commitment required to support FC exceeds the number of periods ahead that choice variables appear in the constraints. In both previous examples we had $L = N$.

While we do not do so here, it is also possible to show that it is not required that the balanced budget rules be constitutionally imposed and hence unchangeable by any future governments. We can make the following, weaker assumption. Suppose that the government is allowed to raise resources via one period bonds, b_{t+1}, with equilibrium price q_t. Define $\mathrm{d}_t = \mathrm{q}_t \mathrm{b}_{t+1}$ as the value of the resources the government raises on the market at time t. If the constitution stipulates that the time-$t+1$ government *must* raise $\bar{d}_{t+1}$ next period, and the time-t government is allowed to choose $\bar{d}_{t+1}$ along with the original time-$t+1$ policy instruments, then again one period of commitment can support FC. Thus budget requirements may be quite flexible, here chosen just one period in advance, while maintaining their commitment enhancing abilities.

Secondly, we consider varying time-to-build, while maintaining the single-period balanced budget assumption. Instead of the standard assumption that investment in period t becomes productive at $t + 1$, we assume that it becomes productive at $t + M$. In this case, we show that M periods of commitment are required to support FC.

Finally, we consider an extension where there is no requirement for balanced budgets, but assuming linear utility from consumption. Even under this assumption, the FC and NC solutions to the model differ, since the FC solution will typically feature zero capital taxes in the long run, and governments without commitment will always be tempted to deviate and increase capital taxes. However, we show that our theorem holds in this model, and a single period of commitment is sufficient to recover the FC solution.

4.5 Comparison with Loose Commitment

The above results allow us to compare optimal policy under LTC with the existing results on optimal policy under Loose Commitment, an assumption that has been explored in two of the economies given above. First, Debortoli and Nunes (2013) study a version of the Lucas and Stokey (1983) economy from Section 4.1 with one period bonds, and endogenous government spending. They show that under Loose Commitment, an equilibrium exists where government debt will always optimally converge to a steady state value independent of initial conditions, even if commitment only lasts on average for one period.

In their economy our theorem holds with a single period of commitment, and thus LTC with one period of commitment is enough to recover the FC solution. However, the FC solution features a long run level of government debt which depends on the government's initial debt position. Hence, perhaps surprisingly, one year of LTC and Loose Commitment which lasts *on average* one year do not lead to similar outcomes.

Second, Debortoli and Nunes (2010) study the capital and labor taxation problem with balanced budgets from Section 4.3. They find that under Loose Commitment the capital tax rate does not converge to zero, as it does under FC. Since LTC supports the FC solution, capital taxes *do* converge to zero under LTC. Hence LTC and Loose Commitment again deliver different results in this model. These results highlight that, once we depart from FC or NC, how we do so can have potentially large implications for the results. This suggests that deviations from full or no commitment might need to be tailored to the exact institutional setting one has in mind.

In general, it should be noted that our equivalence results generalize to different power structures beyond L periods of commitment: it is irrelevant whether the same government is able to commit

beyond that horizon with any probability, because it would anyway want to announce policies consistent with the FC plan. Hence, in our setup, the probability that a government stays in power (or can commit) beyond L periods is irrelevant, as long as commitment for L periods is certain.

5 A new algorithm for FC: policies as state variables

In this section we show that, whenever there exists a degree of commitment, L, such that LTC and FC lead to the same outcomes, it is possible to solve for the time-inconsistent solution to the FC problem using standard dynamic programming tools. This solution coincides with the solution to the LTC problem, and hence uses the LTC states, including the appropriate pre-committed policies, as state variables. This provides an intuitive alternative to using promised utilities or Lagrange multipliers to solve the FC problem, and also allows us to clarify the relationship between the solution to the FC sequence problem and the LTC policy functions. We first give a general outline of the algorithm, and then apply it to the Lucas and Stokey (1983) model.

5.1 Description of algorithm

In Section 3 we proved that whenever the assumptions such that LTC supports FC hold, the FC plan has a recursive form given by (LTC). This recursive form takes as state variables $x_t \equiv (b_t, g_t, \tau_t, ..., \tau_{t+L-1})$, which are precisely the state variables of the LTC problem. The policy and value functions of this recursive form of the FC game can be found by applying standard dynamic programming tools to (LTC).

The only complication is that the state space must be appropriately restricted to rule out choices which violate competitive equilibrium. Note that the statement of (LTC) restricts x_t, and consequently choices for x_{t+1}, to lie in the set X, defined as the set of x_t which lie on some competitive equilibrium path. Rather than having to compute all CE paths and checking whether any x_t lies somewhere on one of them, in Appendix C we provide an algorithm which solves for the set X recursively. This can be done "once and for all" before solving for policy and value functions.

Once the set X is solved for, one can apply standard dynamic programming tools to (LTC) to solve for the "recursified" FC policy functions. Since the LTC policy functions take pre-committed

policies as states, one also has to solve for the initial policies, τ_0^L. This is done by by maximizing the LTC value function, $v(x_0) = v((b_0, g_0, \tau_0, ..., \tau_{L-1}))$, over τ_0^L.

5.2 Illustration: Lucas and Stokey (1983)

This section demonstrates the equivalence of the LTC and FC solutions numerically in the Lucas and Stokey (1983) economy. Since we know that LTC and FC are equivalent, and the solution to FC has been extensively studied, the focus is not on making quantitative statements, but rather qualitatively illustrating the links between the two solutions.

For simplicity we focus on the case of one period bonds with constant government spending, as laid out in the example of Section 2.4. We consider a government who starts with an initial stock of debt $\mathrm{b}_0 > 0$. As previously discussed, the solution to the FC problem in this case involves a tax cut at time 0, followed by higher but constant taxes from time 1 onwards. We denote the optimal time 0 tax rate by $\tau^l_{0,FC}$ and the time 1 and onwards tax rate by τ^l_{FC}. This policy leads to a constant level of debt from time 1 onwards, which we label b_{FC}.

The solution to the LTC problem is given by a policy function $\tau^l_{t+1} = g(\mathrm{b}_t, \tau^l_t)$, and an associated transition for debt from the implementability condition. The policy function is solved for on the set X of values of (b_t, τ^l_t) consistent with at least one competitive equilibrium. This set restricts us to values of initial debt such that the government can actually afford to repay without violating any limits on borrowing or taxes. Figure 1 illustrates the solution, with details on solution method and parametrization relegated to Appendix C.

The left panel shows the set X, represented as an upper and lower limit for debt for a given tax rate: all values between the dashed and solid lines are consistent with at least one competitive equilibrium. Values with high initial debt and low initial taxes are not consistent, since they imply the lowest initial resources for the government, which it may not be feasible to finance.

The right panel plots three slices of the policy function $g\left(\mathrm{b}_t, \tau^l_t\right)$ for different values of b_t. The optimal tax rate tomorrow is increasing in government debt, and decreasing in today's pre-committed tax rate. This is because increasing debt or reducing today's tax both worsen the fiscal position of the government, requiring higher taxes tomorrow to balance the intertemporal budget constraint.

Since our theorem holds in this setup, iterating on this policy function starting from initial

state $\left(\mathrm{b}_0, \tau^l_{0,FC}\right)$ must replicate the FC path for taxes and debt. That is, we must have that $\tau^l_{FC} = g\left(\mathrm{b}_0, \tau^l_{0,FC}\right)$ for time 0, and $\tau^l_{FC} = g\left(\mathrm{b}_{FC}, \tau^l_{FC}\right)$, meaning that the FC plan appears as points which lie on the LTC policy functions. This is shown in Figure 2. The left panel plots $g\left(\mathrm{b}_0, \tau^l_0\right)$, giving a slice of the LTC policy function across different τ^l_0 values for debt equal to the initial value. The cross plots the (independently calculated) FC optimal taxes for $t = 0$ and $t = 1$. The cross lies on the LTC policy function, confirming that the LTC game replicates the FC choice of τ^l_1 if the initial tax is restricted to be $\tau^l_{0,FC}$.

The right panel repeats the exercise for $t = 1$, which is also identical for any $t > 0$. This time the LTC policy function is drawn for debt equal to b_{FC}, which is the value inherited at time 1 for both the LTC and FC policies. Again, the FC cross lies on the LTC line, confirming that the LTC game replicates the FC choice of τ^l_2 since the government inherits a pre-committed choice for τ^l_1 equal to τ^l_{FC}.

6 LTC in a model where equivalence fails: numerical results

In this section, we consider a model in which the conditions for equivalence between LTC and FC fail. In particular, we study the problem of optimal public good provision in a neoclassical growth model, following Klein et al. (2008). We focus on the case in which the public good is financed only through capital taxation and we simplify their model slightly by assuming that labor is inelastically supplied. This serves as a test of LTC in a setting where our theorem doesn't hold, as the missing labor-consumption margin and labor income tax imply that consumption at time t is not pinned down by taxes at t and depends instead on the whole infinite future sequence of policies.

Capital taxation leads to interesting differences between FC and NC outcomes: Klein et al. (2008) show that when the government cannot commit, the steady-state level of the public good (and hence the capital tax rate) is significantly higher than in the FC Ramsey steady-state. Furthermore, we show that lack of commitment leads to a substantial welfare loss (nearly 10% of permanent consumption in steady-state). Hence, we believe it is important to understand what fraction of this welfare loss could be recovered by imposing a "reasonable" degree of commitment in the constitution (e.g., one year).

6.1 A model of public good provision and capital taxation

Preferences of the representative household are represented by the following utility function

$$\sum_{t=0}^{\infty} \beta^t \left[u(\mathrm{c}_t) + z(\mathrm{g}_t)\right] \tag{34}$$

where c_t is the privately-produced consumption good and g_t is the public good, and we assume that $u', z' > 0$, and $u'', z'' < 0$. The budget constraint of the household is

$$\mathrm{c}_t + \mathrm{k}_{t+1} = \mathrm{w}_t \mathrm{l}_t + \mathrm{k}_t \left[1 + \mathrm{r}_t(1 - \tau_t^k)\right]. \tag{35}$$

Output is produced with production function $\mathrm{y}_t = \mathrm{k}_t^{\alpha} \mathrm{l}_t^{1-\alpha}$, with labor inelastically supplied at $\mathrm{l}_t = 1$. Firms profit maximization implies that the pre-tax return on assets is given by $\mathrm{r}_t = \alpha \mathrm{k}_t^{\alpha-1} - \delta$ and the wage by $\mathrm{w}_t = (1-\alpha)\mathrm{k}_t^{\alpha}$. The resource constraint of the economy reads

$$\mathrm{c}_t + \mathrm{k}_{t+1} + \mathrm{g}_t = \mathrm{k}_t^{\alpha} + (1-\delta)\mathrm{k}_t, \tag{36}$$

and the government balanced-budget constraint is[27]

$$\tau_t^k (\alpha \mathrm{k}_t^{\alpha} - \delta \mathrm{k}_t) = \mathrm{g}_t. \tag{37}$$

The competitive equilibrium is fully characterized by (36) and the following Euler equation for capital, where we substitute taxes from (37).

$$u'(\mathrm{c}_t) = \beta u'(\mathrm{c}_{t+1}) \left(1 + \alpha \mathrm{k}_{t+1}^{\alpha-1} - \delta - \frac{\mathrm{g}_{t+1}}{\mathrm{k}_{t+1}}\right) \tag{38}$$

To see why the equivalence between LTC and FC does not hold in this model, notice that a key difference with respect to the version presented in Section 4 is the absence of the intratemporal labor-consumption margin, (30), which determines the only level of consumption consistent with household optimality, given the only level of hours worked consistent with government balanced budget. With exogenous labor, consumption and investment at time t are not uniquely pinned down by the tax rate at time t, and depend instead on the whole infinite sequence of future tax rates.

In Appendix D we provide the first order conditions of the FC Ramsey plan as well as a characterization of the smooth NC equilibrium as analyzed by Klein et al. (2008). In the case of LTC, we

[27] Differently from Section 4, here we allow for a deduction for depreciation before capital taxes are charged.

assume that governments have one period of commitment, which in our calibration will correspond to a year.[28] Thus, the natural state variables of the Markov perfect LTC equilibrium are $(\mathrm{k}_t, \mathrm{g}_t)$, that is, the government at time t inherits an aggregate capital stock as well as a fiscal plan to be enacted at time t.[29] In a symmetric equilibrium, all governments play the same policy function, $\mathrm{g}_{t+1} = \tilde{g}^{LTC}(\mathrm{k}_t, \mathrm{g}_t)$. Details of the recursive formulation of the game are relegated to Appendix D, which also includes a description of a simple global algorithm to solve for smooth LTC equilibria based on projection with third-order polynomials.

6.2 Calibration

We follow the calibration choices of Klein et al. (2008) in the interest of comparison, with the difference that we set hours worked equal to an exogenous constant. Table 1 reports the parameter values. One period in the model corresponds to a year, hence we study the case where governments can commit to fiscal policy for a single fiscal year. We parametrize utility as follows: $u(\mathrm{c}) = \log(\mathrm{c})$ and $z(\mathrm{g}) = D\log(\mathrm{g})$.[30]

6.3 Steady-state comparison: FC, NC, LTC

In Table 2 we compare steady-state allocations, policies and welfare in three economies: FC, NC and LTC.[31] The LTC government internalizes the distortionary effect of capital taxes promised for the following period and accordingly lowers them relative to the NC government. However, tax rates beyond one period ahead are out of the LTC government's control, just like taxes at $t+1$ for the NC government. Because the whole infinite sequence of future tax rates matters for the allocation at t in this model, the LTC equilibrium does not coincide with FC, and is characterized by higher

[28] Klein and Ríos-Rull (2003) also assume one year of commitment to capital taxes, but no commitment to labor taxes, and compute linear approximations of optimal policy in a model with shocks to productivity and spending. Differently from them, we consider a deterministic economy with endogenous g, abstract from labor taxes and focus on a global solution that allows to characterize transitional dynamics after a constitutional reform.

[29] Thanks to the balanced-budget, we could equivalently use the tax rate as a state variable.

[30] With linear u, we would recover our equivalence between LTC and FC outcomes.

[31] The assumption of exogenous labor drives the difference between the statistics reported in our NC column and the corresponding column in Table 1 of Klein et al. (2008). We have verified that our method delivers the same steady-state results as theirs in the case of NC and endogenous labor supply.

taxes. Hence, LTC taxes and public good provision in steady-state are set at an intermediate level between FC and NC.[32]

This result on policies leads to an intermediate steady-state level of capital, consumption and output. Notice that out of a given amount of output produced, governments can affect how these resources will be devoted to private consumption, public consumption and investment. Interestingly, we find that most of the difference in the allocation between LTC and NC is driven by the fact that a larger fraction of output is devoted to investment under LTC. This extra investment is financed almost entirely by reducing the fraction of public spending, while the fraction of output devoted to private consumption is remarkably similar across all three economies.

We also compute the welfare losses from FC to NC (and LTC) as the fraction of steady-state consumption that would make the representative household indifferent between living in these different economies and living in the FC economy. Going from FC to NC is equivalent to a drop in permanent consumption of 9.6%. Approximately a third of this loss can be recovered by imposing a single year of commitment to fiscal policy.

6.4 Transitional dynamics

Using our global solution method, we can compute the transitional dynamics of the LTC economy starting from an arbitrary initial capital stock and policy. We choose to compute the transition from the steady-state associated with the NC regime. This exercise may shed light on the benefits of implementing a limited degree of commitment to fiscal policy in the constitution in economies where governments lack credibility.

Figure 3 shows the paths of capital, private consumption, public consumption and tax rate. The solid red line shows the transition of interest, the dashed blue line illustrates the counterfactual NC steady-state. Interestingly, the optimal transition path involves a non-monotonic path for the size of the government. We observe a large fiscal adjustment (cut in g_t) at the beginning, followed by an increase in the provision of the public good. This is because the first LTC government inherits a ratio between public and private consumption that is "too high". By partially internalizing

[32]In all three economies, taxes are substantially higher than in the US economy. This is because the only tax base in the model is capital income net of depreciation.

the distortionary effect of capital taxes, the LTC government decides to decrease the size of the government (both in terms of spending and tax rate) at the beginning in order to foster capital accumulation. The overall welfare benefit of this reform is equal to 1.7% of permanent consumption, stemming from capital accumulation equal to 8% of the initial stock, and an increase in output of 4.4%.

This economy is very stylized, hence we do not wish to draw strong quantitative conclusions from this exercise. However, the significant welfare gain in this simple economy suggests that creating institutional devices that induce even a small degree of commitment to fiscal policy may lead to non-trivial benefits in economies characterized by lack of commitment.

7 Conclusion

In this paper we have studied optimal policy in economies where successive one-period lived governments formulate plans for a finite horizon. We have emphasized a key condition on the mapping from policy instruments to allocations. If this condition is satisfied in a given model, then this Limited-Time Commitment game with (sufficiently long) finite commitment leads to the same outcomes that would arise if there were a single government at time 0, endowed with Full Commitment into the infinite future.

This is indeed the case for a number of economies that have been studied in the fiscal policy literature. In this sense, we have provided a case for assuming Full Commitment in those models: even with a lighter commitment assumption we find the same results. This is also comforting from a practical perspective, given that solving for NC equilibria, particularly in models with government debt, can lead to numerical issues, as well as issues of multiplicity of equilibria. Additionally, we relate the length of commitment required to support FC to economic features, which could allow disciplining of the appropriate equilibrium concept based on the model at hand.

In a model of labor taxes without capital, we require commitment equal to the length of the longest maturity bond. In a model of capital taxation with balanced budgets, we require commitment equal to the length of time over which the budget must be balanced (or the time to build of capital, if longer). In this sense, this paper uncovers a role for balanced-budget rules in building commitment. In a stochastic economy, a balanced-budget rule imposes costs in terms of lack of tax smoothing. We

leave for future work a quantitative analysis of the trade-off between such costs and the commitment benefits of balanced-budget rules in a business-cycle model.

Another result of our analysis is that once we start making assumptions that limit the government's commitment technology, the details of how we do so can be quite important: we obtain equivalence with Full Commitment in cases where the Loose Commitment approach (probabilistic commitment into the infinite future) would lead to outcomes more similar to No Commitment.

We have also provided a new algorithm to solve for optimal policy with FC using pre-committed policies as state variables. This algorithm works whenever our equivalence theorem holds and constitutes an intuitive alternative to using promised utilities or Lagrange multipliers as currently done in the literature.

Finally, we have studied a model of capital taxation where LTC and FC are not equivalent and we have shown that introducing a single year of commitment to capital taxes in the constitution induces substantial welfare gains relative to a regime of NC. In this sense, this paper makes a case for designing institutional features that induce realistic degrees of commitment to fiscal policy.

In this paper we have worked with deterministic economies, and we leave a general characterization of optimal policy with Limited-Time Commitment in stochastic economies for future work. However, we conjecture that our main equivalence result will survive in stochastic models, provided that governments can commit to finite sequences of state-contingent policy instruments.

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A Appendix to Section 3

Lemma A1. *For any $x_0 \in X$:*

1. *The set of competitive equilibrium paths, $\Pi(x_0)$, is non empty*
2. *For all $\mathbf{x} \in \Pi(x_0)$, $\lim_{n\to\infty} \sum_{t=0}^{n} \beta^t F(x_t, x_{t+1})$ exists, although it may be plus or minus infinity.*

Proof. Point 1 follows trivially from the non-emptiness of Γ for any $x \in X$. For point 2, note that **Lemma 2** established that any $\mathbf{x} \in \Pi(x_0)$ has a unique associated path $\mathbf{y} \in \Pi^*(b_0, g_0)$ for some $(b_0, g_0) \in B^*$. By **Assumption 1*** we know that for this $\mathbf{y}$, $\lim_{n\to\infty} \sum_{t=0}^{n} \beta^t r(c_t, b_t, g_t, \tau_t)$ exists, although it may be plus or minus infinity. Given that we defined F as $F(x_t, x_{t+1}) = r(c_t, b_t, g_t, \tau_t)$, it must also be that $\lim_{n\to\infty} \sum_{t=0}^{n} \beta^t F(x_t, x_{t+1})$ exists. $\square$

Proof of Lemma 1. $\Gamma : X \mapsto X$ is defined s.t. x_{t+1} is on a competitive equilibrium plan given x_t iff $x_{t+1} \in \Gamma(x_t)$. To show that such a definition is possible we first need to show that we can check if x_{t+1} is on at least one competitive equilibrium plan given x_t without knowledge of $x_{t+2}, x_{t+3}, ...$ Since $x_{t+1} \in X$, we know that it lies on at least one competitive equilibrium plan from $t+1$ onwards, from the set $\Pi^*(b_{t+1}, g_{t+1})$. (x_t, x_{t+1}) then lies on at least one competitive equilibrium plan from t onwards, from the set $\Pi^*(b_t, g_t)$, iff $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$. **Assumption 3*** implies that if (x_t, x_{t+1}) does lie on any competitive equilibrium plans, they must all have the same values of y_t, b_{t+1}, and the variables which are problematic from the perspective of time t. These are all the variables needed to check whether $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$, and thus we can establish whether x_{t+1} lies on at least one competitive equilibrium plan given x_t with knowledge of only (x_t, x_{t+1}).

Since Γ^* and the relationship between (x_t, x_{t+1}) and the variables it pins down are both time invariant, defining Γ without time dependence is possible. $\Gamma(x_t)$ is non-empty for all $x_t \in X$ because being in X implies that x_t lies on at least one competitive equilibrium plan, $\mathbf{y}$. The values $(y_{t+1}, ..., y_{t+L})$ from this plan define a value for x_{t+1} on a competitive equilibrium plan. $\square$

Proof of Lemma 2. For the first statement: For all $(b_0, g_0) \in B^*$, each $\mathbf{y} \in \Pi^*(b_0, g_0)$ defines a unique path $\mathbf{x}$, including its initial element x_0. Since $\mathbf{y}$ is a competitive equilibrium, we know that $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$, and hence, by the definition of Γ, that $x_{t+1} \in \Gamma(x_t)$, for all $t = 0, 1, ...$, and that $x_0 \in X$. Thus $\mathbf{x} \in \Pi(x_0)$. For the converse: For all $x_0 \in X$, each $\mathbf{x} \in \Pi(x_0)$ means that

$x_{t+1} \in \Gamma(x_t)$ for all $t = 0, 1, ...$ The definition of Γ means that each pair (x_t, x_{t+1}) along the path defines a unique sequence $(y_t, y_{t+1}, ..., y_{t+N})$ which satisfies $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$. This is thus a competitive equilibrium path $\mathbf{y} \in \Pi^*(b_0, g_0)$, with $(b_0, g_0) \in B^*$ taken from y_0. $\square$

Proof of Lemma 3. We established in **Lemma 2** the equivalence of $\mathbf{x}$ and $\mathbf{y}$ paths. Suppose that for some $(b_0, g_0) \in B^*$ we had $V^*(b_0, g_0) > \sup_{(\tau_0,...,\tau_{L-1}) \in Q(b_0,g_0)} V((b_0, g_0, \tau_0, ..., \tau_{L-1}))$, where the left hand side supremum is achieved with the path $\mathbf{y}$ and the right hand side by the path $\mathbf{x}'$. The path $\mathbf{x}$ defined by $\mathbf{y}$ is a competitive equilibrium and delivers higher utility, contradicting that the right hand side is the supremum. The equivalent argument applies in the opposite direction, leaving equality as the only possibility. $\square$

Proof of Lemma 4. The following four statements follow from Theorems 4.2-4.5 in Stokey and Lucas (1989). We require that their Assumptions 4.1 and 4.2 hold, which we proved in **Lemma 1** and **Lemma A1**.

1. The function V satisfies (LTC).

2. Let $\mathbf{x}^* \in \Pi(x_0)$ be a competitive equilibrium plan that attains the supremum in (MP) for initial state x_0. Then

$$V(x_t^*) = F(x_t^*, x_{t+1}^*) + \beta V(x_{t+1}^*), \quad t = 0, 1, 2, ... \tag{P}$$

3. If v is a solution to (LTC) and satisfies

$$\lim_{t\to\infty} \beta^t v(x_t) = 0, \quad \forall \mathbf{x} \in \Pi(x_0), \forall x_0 \in X \tag{BC}$$

then $v = V$.

4. Let $\mathbf{x}^* \in \Pi(x_0)$ be a competitive equilibrium plan for initial state x_0 satisfying (P) and with

$$\lim_{t\to\infty} \beta^t V(x_t^*) \leq 0$$

Then $\mathbf{x}^*$ attains the supremum in (MP) for initial state x_0.

The first two items of the lemma prove that MP solves the LTC problem: the MP value function $V(x)$ satisfies the LTC recursion, as do the generated optimal plans. The second two items, combined with

Assumption 4* can then be used to show that these are the unique function and plans which solve the LTC recursion. **Assumption 4*** states that the return function is bounded, which implies that there is a finite $\bar{F} < \infty$ for which $|F(x, y)| < \bar{F}$ for any $x, y \in X$. This implies that $|v(x)| \leq \bar{F}/(1-\beta)$ for any candidate solution to LTC, including the MP solution, V. This bound, combined with $\beta < 1$, implies that 1) the boundedness condition in point 3 is satisfied for any candidate solution to LTC, meaning that all solutions must be V, and 2) the boundedness condition in point 4 is satisfied for any plan, meaning that any optimal plans in the LTC game must be an optimal plan in MP. □

B Appendix to Section 4

B.1 Boundedness restrictions

The boundedness restriction in **Assumption 4*** is satisfied in the applications as long as there are no competitive equilibria leading to (positive or negative) infinite value, which is a relatively weak restriction. Competitive equilibrium places an upper bound on period utility, since consumption must ultimately be produced according to the economy's production technology, which is bounded every period if we assume an upper bound for labor supply and capital.

To ensure a lower bound on period utility is less straightforward, since governments in these models may be able to "shut down" the economy by issuing arbitrarily high taxes. This can push consumption towards zero, which leads utility to tend to $-\infty$ without bound with, for example, CRRA utility over consumption. This can be ruled out by an appropriate upper bound on taxation, such as requiring that the government must remain to the left of the peak of the dynamic Laffer curve, or by an arbitrary lower bound on utility. These bounds can always be chosen to be non-binding along an optimal path.

B.2 Additional model 1: Multi-period balanced budget in the capital tax model

In this section we consider a government who faces the constraint that she must balance her budget every M periods. There are many ways to implement this which lead to LTC supporting FC, and we illustrate one method here. In particular, we suppose that the government can issue one-period

bonds, denoted b_t, which are priced according to the agent's Euler equation:

$$\mathrm{q}_t u_{\mathrm{c}_t} = \beta u_{\mathrm{c}_{t+1}} \tag{39}$$

Where $u_{\mathrm{c}_t} \equiv u_{\mathrm{c}}(\mathrm{c}_t, \mathrm{g}_t, \mathrm{l}_t)$. The government's budget is now:

$$\mathrm{q}_t \mathrm{b}_{t+1} + \left[\alpha\tau_t^k + (1-\alpha)\,\tau_t^l\right](\mathrm{v}_t \mathrm{k}_t)^\alpha\, \mathrm{l}_t^{1-\alpha} = \mathrm{g}_t + \mathrm{b}_t \tag{40}$$

Substituting in the bond Euler gives:

$$\beta\frac{u_{\mathrm{c}_{t+1}}}{u_{\mathrm{c}_t}}\mathrm{b}_{t+1} + \left[\alpha\tau_t^k + (1-\alpha)\,\tau_t^l\right](\mathrm{v}_t \mathrm{k}_t)^\alpha\, \mathrm{l}_t^{1-\alpha} = \mathrm{g}_t + \mathrm{b}_t \tag{41}$$

We implement the balanced budget assumption by assuming that the government cannot issue bonds once every M periods. Give each period an index $m_t \in \{1, 2, ..., M\}$ denoting its position in the cycle, with $m_t = M$ denoting the last period of the cycle (where the government can't issue debt) and $m_t = 1$ denoting the first (where there is thus no inherited debt to repay). To fix ideas, consider a model where one period is a quarter, and the government must balance her yearly budget. This means that $M = 4$, and the government cannot issue any bonds in the fourth quarter of every year.

The rest of the model equations are as in the baseline model where the government must balance the budget every period: (29), (30), (31) and (32), to which we add the government budget, (41), and the restriction that $\mathrm{b}_{t+1} = 0$ if $m_t = M$. We now prove that we can support FC with LTC with $L = M$ periods of commitment.

To prove **Assumption 3***, we need to prove that given the natural state $(\mathrm{k}_t, \mathrm{b}_t)$, if we fix $(\tau_t^k, \tau_t^l, \mathrm{g}_t, ..., \tau_{t+M}^k, \tau_{t+M}^l, \mathrm{g}_{t+M})$, then (i) we pin down all of $y_t = (\mathrm{k}_t, \mathrm{b}_t, \mathrm{c}_t, \mathrm{l}_t, \mathrm{v}_t, \tau_t^k, \tau_t^l, \mathrm{g}_t)$, of which we only need to check $(\mathrm{c}_t, \mathrm{l}_t, \mathrm{v}_t)$, and $(\mathrm{k}_{t+1}, \mathrm{b}_{t+1})$, and 2) the problematic variables $(\mathrm{c}_{t+1}, \mathrm{l}_{t+1}, \mathrm{v}_{t+1})$ are uniquely pinned down given $(\mathrm{k}_{t+1}, \mathrm{b}_{t+1}, \tau_{t+1}^k, \tau_{t+1}^l, \mathrm{g}_{t+1}, ..., \tau_{t+M}^k, \tau_{t+M}^l, \mathrm{g}_{t+M})$.

This has do be done separately for each position in the cycle, but the procedure is similar in all cases. First consider a period where $m_t = 1$, at the beginning of the cycle. To check point (i), we can forward (41) from t to $t + M - 1$ to yield:

$$\sum_{s=0}^{M-1} u_{\mathrm{c}_{t+s}}\left[\alpha\tau_{t+s}^k + (1-\alpha)\,\tau_{t+s}^l\right](\mathrm{v}_{t+s}\mathrm{k}_{t+s})^\alpha\, \mathrm{l}_{t+s}^{1-\alpha} = \sum_{s=0}^{M-1} u_{\mathrm{c}_{t+s}}\mathrm{g}_{t+s} \tag{42}$$

Combining this with

- M resource constraints, (29), from t to $t+M-1$
- M labor FOCs, (30), from t to $t+M-1$
- M utilization FOCs, (31), from t to $t+M-1$
- $M-1$ capital Euler equations, (32), from t to $t+M-2$

gives $4 \times M$ equations in $4 \times M$ unknowns, $\{\mathrm{c}_{t+s}, \mathrm{l}_{t+s}, \mathrm{v}_{t+s}, \mathrm{k}_{t+s+1}\}_{s=0}^{M-1}$. While the number of equations does equal the number of unknowns, given the nonlinearity of the system we are unable to prove that this system admits a unique solution, and must maintain sufficient functional form restrictions as an additional assumption to be checked case by case. Under this assumption, this system thus pins down $(\mathrm{c}_t, \mathrm{l}_t, \mathrm{v}_t)$, and $(\mathrm{k}_{t+1}, \mathrm{b}_{t+1})$, which were required for point (i).

To check point (ii) forward (41) from $t+1$ to $t+M-1$, given $(\mathrm{k}_{t+1}, \mathrm{b}_{t+1})$, and apply the same procedure using the same equations from $t+1$ to $t+M-1$ to solve for $(\mathrm{c}_{t+1}, \mathrm{l}_{t+1}, \mathrm{v}_{t+1})$. A similar procedure can be applied for periods at different points in the cycle.

B.3 Additional model 2: Time-to-build

In this section we consider an extension of the model of Section 4.3 with single-period balanced budgets but M period time-to-build. By M period time-to-build, we mean that investment today, i_t, creates capital which becomes productive at time $t+M$:

$$\mathrm{k}_{t+M} = (1-\delta)\mathrm{k}_{t+M-1} + \mathrm{i}_t \tag{43}$$

Note that k_{t+M-1} is predetermined at time t. In this case the agent's Euler equation for capital becomes:

$$u_{c_t} = \beta u_{c_{t+1}}(1-\delta(\mathrm{v}_{t+1})) + \beta^M u_{c_{t+M}} \alpha \left(\mathrm{v}_{t+M}\mathrm{k}_{t+M}\right)^{\alpha-1} \mathrm{l}_{t+M}^{1-\alpha} \left(1-\tau_{t+M}^k\right) \tag{44}$$

All other equations of the model remain the same, apart from the resource constraint which now reads $\mathrm{c}_t + \mathrm{k}_{t+M} - (1-\delta)\,\mathrm{k}_{t+M-1} + \mathrm{g}_t = \mathrm{k}_t^\alpha \mathrm{l}_t^{1-\alpha}$. The problematic variables are now c_{t+1}, l_{t+1}, v_{t+1}, c_{t+M}, l_{t+M}, and v_{t+M}, as well as the policies g_{t+1}, g_{t+M}, and τ_{t+M}^k. In the notation of our general formulation we have $N = M$, with variables up to M periods ahead appearing in the constraints.

As in the original model with one period time-to-build, given $\left(\mathrm{k}_t, \tau_t^k, \tau_t^l, \mathrm{g}_t\right)$, equations (29), (30), (31) and (33) still form a system of four (non-linear) equations in four unknowns, namely

(c_t, l_t, v_t, k_{t+1}), which admit a unique solution under the original assumptions. However, note that to satisfy the second part of **Assumption 3*** we need to be able to pin down $(c_{t+1}, l_{t+1}, v_{t+1}, c_{t+M}, l_{t+M}, v_{t+M})$ given k_{t+1} and the policies predetermined from the time-$t+1$ government's perspective.

One period of commitment is no longer enough to achieve this, since $(k_{t+1}, \tau^k_{t+1}, \tau^l_{t+1}, g_{t+1})$ will only pin down $(c_{t+1}, l_{t+1}, v_{t+1})$. However, by the same logic $(k_{t+1}, \tau^k_{t+1}, \tau^l_{t+1}, g_{t+1}, ..., \tau^k_{t+M}, \tau^l_{t+M}, g_{t+M})$ will pin down all of the required problematic variables. Thus, M periods of commitment are sufficient to recover FC in this extension.

B.4 Additional model 3: Unbalanced budget in the capital tax model

In this section we consider an extension of the capital taxation model where the government is able to borrow and lend from the household using a one-period bond. Thus we replace the government's balanced equation with (41) from the previous section, allowing for government debt, b_t pricing according to the household's Euler equation.

In the general notation of Section 3 we have $b_t = (k_t, b_t)$, $c_t = (c_t, l_t, v_t)$, $p_t = q_t$, and $\tau_t = (\tau^k_t, \tau^l_t, g_t)$. There are no exogenous states (g_t is empty). The transition correspondence, Γ^*, is defined by (29), (30), (31), (32), and (41).

It is possible to prove that our theorem holds in the special case of linear utility from consumption. In particular, consider a version of the model with constant government spending, g, which is purely wasteful, a utility function $u(c_t, g_t, l_t) = c_t - v(l_t)$, and no capital utilization margin, giving $v_t = 1$. The function $v(l_t)$ is assumed to satisfy the usual conditions $v_l > 0$ and $v_{l,l} > 0$. The equations of the model are now:

$$c_t + k_{t+1} - (1-\delta)\, k_t + g_t = k_t^{\alpha} l_t^{1-\alpha}. \tag{45}$$

$$v'(l_t) = \left(1 - \tau^l_t\right)(1-\alpha) k_t^{\alpha} l_t^{-\alpha} \tag{46}$$

$$\beta b_{t+1} + \left[\alpha \tau^k_t + (1-\alpha)\, \tau^l_t\right] k_t^{\alpha} l_t^{1-\alpha} = g + b_t \tag{47}$$

$$1 = \beta \left[\alpha k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} \left(1 - \tau^k_{t+1}\right) + 1 - \delta\right] \tag{48}$$

Since there is no utilization margin we also add the constraint that capital taxes cannot exceed an upper bound $\bar{\tau}^k$. Even with linear utility from consumption, there is still a meaningful distinction between the FC and NC solutions to this model. A government with FC will choose to have the

time-0 capital tax at the maximum level, and then set capital taxes from time 1 onwards to zero.[33] Labor taxes will be constant from period 1 onwards. A government with NC, on the other hand, will have the temptation to tax capital once it is installed, and is going to set capital taxes to the maximum level for a long time, potentially forever.

To see that LTC can support FC in this special case, note that the only problematic variable is now l_{t+1}. In the notation of our general formulation we have $N = 1$, with variables one period ahead appearing in the constraints. We need to show that **Assumption 3***, holds in this model for $L = 1$. In other words, we need to show that given the natural state $(\mathrm{k}_t, \mathrm{b}_t)$, if we fix $(\tau_t^k, \tau_t^l, \tau_{t+1}^k, \tau_{t+1}^l)$, then 1) we pin down all of $y_t = (\mathrm{k}_t, \mathrm{c}_t, \mathrm{l}_t, \tau_t^k, \tau_t^l)$, of which we only need to check $(\mathrm{c}_t, \mathrm{l}_t)$, and $(\mathrm{k}_{t+1}, \mathrm{b}_{t+1})$, and 2) the problematic variable l_{t+1} is fixed given $(\mathrm{k}_{t+1}, \mathrm{b}_{t+1}, \tau_{t+1}^k, \tau_{t+1}^l)$.

To see that this is the case note that, given k_t and τ_t^l, (46) pins down a unique l_t, defining a function $\mathrm{l}_t = l(\mathrm{k}_t, \tau_t^l)$. Combined with (48) and the time-t government's choice of $(\tau_{t+1}^k, \tau_{t+1}^l)$ this uniquely pins down $(\mathrm{k}_{t+1}, \mathrm{l}_{t+1})$. Finally, the resource constraint, (45) uniquely pins down c_t, and the government's budget, (47), uniquely pins down b_{t+1}.

The economic reason that one period of commitment can sustain FC in this model is that it features one period time-to-build. It is simple to prove, as we did in the second extension, that with M period time-to-build M periods of commitment are required to support FC in this model.

B.5 Arbitrary initial conditions in the capital tax model

In the special case of the capital tax model with linear utility from consumption for which our theorem holds, we can also prove that the equilibrium of the LTC game converges to a different FC solution if a generic time-t government inherits "incorrect" policies. Recall that in this model the FC solution for $t > 0$ features zero capital taxes and constant labor taxes.

To prove this, it is convenient to combine the competitive equilibrium constraints into a single

[33]Straub and Werning (2015) show that in the presence of an upper bound on capital taxation, it is possible that the Chamley-Judd result that capital taxes converge to zero does not hold for sufficiently high initial government debt, and the upper bound optimally binds asymptotically. We assume that initial government debt is low enough to avoid this case.

implementability constraint:

$$\frac{1}{\beta}\mathrm{k}_t + \mathrm{b}_t = \mathrm{c}_t - v_{l,t}\mathrm{l}_t + \mathrm{k}_{t+1} + \beta \mathrm{b}_{t+1} \tag{49}$$

This constraint combines all the time-t constraints except for the capital Euler equation, and also incorporates the time-$t-1$ Euler equation.[34] We can write the government's problem recursively. Note that we can use $(\mathrm{b}_t, \mathrm{k}_t, \mathrm{l}_t)$ as the only states since we can use the time-t labor condition, (46), and $t-1$ Euler, (48) to infer what time-t taxes they imply, instead of holding the taxes as additional states.

$$W(\mathrm{b}_t, \mathrm{k}_t, \mathrm{l}_t) = \max_{\mathrm{b}_{t+1}, \mathrm{k}_{t+1}, \mathrm{l}_{t+1}} \mathrm{k}_t^{\alpha}\mathrm{l}_t^{1-\alpha} + (1-\delta)\,\mathrm{k}_t - \mathrm{g} - \mathrm{k}_{t+1} - v(\mathrm{l}_t) + \beta W(\mathrm{b}_{t+1}, \mathrm{k}_{t+1}, \mathrm{l}_{t+1}) \tag{50}$$

where the maximization is subject to (49). Denote by λ_t the multiplier on (49), then the capital, bond and labor first order conditions give respectively:

$$1 - \lambda_t = \beta\left(\alpha \mathrm{k}_t^{\alpha-1}\mathrm{l}_t^{1-\alpha} + 1 - \delta\right) - \lambda_{t+1} \tag{51}$$

$$\lambda_t = \lambda_{t+1} \tag{52}$$

$$(1-\alpha)\mathrm{k}_{t+1}^{\alpha}\mathrm{l}_{t+1}^{-\alpha} - v'(\mathrm{l}_{t+1}) - \lambda_{t+1}(v''(\mathrm{l}_{t+1})\mathrm{l}_{t+1} + v'(\mathrm{l}_{t+1})) = 0 \tag{53}$$

We can combine the capital and bond first order conditions to give:

$$1 = \beta\left(\alpha \mathrm{k}_t^{\alpha-1}\mathrm{l}_t^{1-\alpha} + 1 - \delta\right) \tag{54}$$

This is just the household's capital Euler equation with zero capital taxes. Hence we have shown that regardless of the initial condition at time-t, a government with LTC will always immediately set $\tau_{t+1}^k = 0$. Labor taxes will be constant from period $t+1$ onwards because both the multiplier and capital in (53) are constant, implying constant hours, and hence constant labor taxes. The level these constant labor taxes must be set at can be solved for by iterating the government's budget constant forward and imposing transversality. These constant labor taxes must be the solution to a FC game for a different value of the initial endogenous state variables.

[34] Thus the constraint requires that the time-t capital tax is consistent with the optimal choice of k_t made one period before. Hence the following discussion applies to any deviation of $t+1$ policy from the FC plan which is announced at time-t, not surprise deviations. The result that we converge to another FC plan also holds for surprise deviations, but the convergence simply takes one period longer.

C Numerical appendix: solving models where the LTC-FC equivalence holds

C.1 Algorithm to construct feasible set, X

In this section we construct a procedure to calculate the set X, on which the LTC problem is defined. Recall that X was defined as:

$$X = \left\{x \in B \times G \times T^L : x \text{ lies on at least one path } \mathbf{y} \in \Pi^*(b_0, g_0) \text{ for some } (b_0, g_0) \in B^*\right\} \tag{55}$$

The procedure is iterative, and based on Phelan and Stacchetti's (2001) procedure for solving for the value correspondence in their setup. First we define a transition correspondence:

Definition 6. *$\tilde{\Gamma} : B \times G \times T^L \mapsto B \times G \times T^L$ is defined such that $x_{t+1} \in \tilde{\Gamma}(x_t)$ iff there exists a $(y_{t+1}, ..., y_{t+N}) \in Y^N$ such that 1) $(y_{t+1}, ..., y_{t+N})$ contains the problematic elements $\{b_{t+s}, c_{t+s}, p_{t+s}\}_{s=1}^N$ and b_{t+1} uniquely pinned down by (x_t, x_{t+1}), and 2) $(y_{t+1}, ..., y_{t+N}) \in \Gamma^*(y_t)$.*

Note that this is closely related to the correspondence Γ, but is defined on the more general set $B \times G \times T^L$, since we do not know the set X at this point. Now, for a generic set $X_n \in B \times G \times T^L$, define the mapping over sets:

$$B(X_n) = \left\{x \in B \times G \times T^L : \text{there exists } x' \in \tilde{\Gamma}(x) \text{ s.t. } x' \in X_n\right\} \tag{56}$$

$B(X_n)$ is the set of $x_t = (b_t, g_t, \tau_t^L)$ such that it is possible to find a $x_{t+1} = (b_{t+1}, g_{t+1}, \tau_{t+1}^L)$ which satisfies today's CE constraints (given x_t) and leaves $x_{t+1} \in X_n$. We first prove two preliminary lemmas:

Lemma 5. $X = B(X)$

Proof. Suppose that $X \supset B(X)$ strictly. Then there exists some $\hat{x} \in B(X)$ and $\hat{x} \notin X$ such that there exists $x' \in \tilde{\Gamma}(\hat{x})$ with $x' \in X$. Since x' is on a CE path and is reachable from $\hat{x}$, $\hat{x}$ must also be on a CE path which features $\hat{x}$ and x' as its first and second elements. This contradicts $\hat{x} \notin X$. Now suppose that $X \subset B(X)$ strictly. Then there exists some $\hat{x} \in X$ such that there exists no $x' \in \tilde{\Gamma}(\hat{x})$ with $x' \in X$. Thus $\hat{x}$ cannot be on a CE path, contradicting $\hat{x} \in X$. Thus it must be that $X = B(X)$. □

Lemma 6. *If $X_{n+1} \subset X_n$, then $B(X_{n+1}) \subset B(X_n)$.*

Proof. Consider a generic $x \in X_{n+1}$. To be in $B(X_{n+1})$ there must exist an x' such that $x' \in X_{n+1}$ and $x' \in \tilde{\Gamma}(x)$. Since $X_{n+1} \subset X_n$, we must have $x' \in X_n$, and hence also $x \in B(X_n)$. Since this is true for any $x \in X_{n+1}$, we have $B(X_{n+1}) \subset B(X_n)$. $\square$

We are now ready to state the main result:

Lemma 7. *Suppose that $X_0 \supset X$, and $B(X_0) \subset X_0$. Define the recursion $X_{n+1} = B(X_n)$. Then $\lim_{n\to\infty} X_n = X$.*

Proof. The proof is recursive. 1) $X_1 = B(X_0) \subset X_0 \Rightarrow B(X_1) \subset B(X_0) \Rightarrow X_2 \subset X_1$. Carrying this on, we find that $X_n \subset X_{n-1} \subset ... \subset X_0$ for any n. 2) Also, $X \subset X_0 \Rightarrow B(X) \subset B(X_0) \Rightarrow X \subset X_1$. This inductively implies that $X \subset X_n$ for any n. Combining points 1 and 2 gives $X \subset X_n \subset X_{n-1} \subset ... \subset X_0$. Since this sequence of sets is decreasing it has a limit in the sense of set inclusion: $X_\infty \equiv \lim_{n\to\infty} X_n$. By a simple limit argument, $X_\infty = X$. $\square$

This recursion is thus guaranteed to converge to X for an appropriately chosen X_0. A simple example of an X_0 which satisfies the required conditions is $X_0 = B \times G \times T^L$.

C.2 Lucas and Stokey (1983) economy

Utility is parameterized as $u(\mathrm{c}, \mathrm{l}) = \mathrm{c}^{1-\sigma}/(1-\sigma) - D\mathrm{l}^{1+\eta}/(1+\eta)$, specialized to $\sigma = \eta = 1$. We set $\beta = 0.96$ as standard. We calibrate g, b_0 and D to a fictional steady state were taxes to be constant. We target $\mathrm{l} = 1/3$ by appropriately choosing D, and set g to 20% of output. We target debt to GDP of around 60% and choose b_0 accordingly, leading to required constant taxes of around 22% in the fictional steady state.

The LTC game is solved using value function iteration. We discretize the state, (b, τ^l), and maximize over $(\tau^l)'$ using a grid search procedure, with b' solved for explicitly using the implementability constraint. Future values off the grid are interpolated using splines. The set X is solved for using the iterative procedure developed in the paper. We solve for explicit maximum and minimum feasible b values on the τ^l grid. During the value function iteration, we restrict the government from making $(\tau^l)'$ choices which would lead $((\tau^l)', \mathrm{b}')$ to be outside of X.

To choose grids for the LTC game, we first solve the FC problem starting from b_0. We then take the maximum and minimum values for the grids for b and τ^l to be 5% above and below the largest values observed along the FC path respectively.

D Public good and capital taxation: characterization of FC and NC and solution algorithm for LTC

D.1 Full Commitment

Let λ_t and γ_t be the Lagrange multipliers on (36) and (38) respectively. Starting from a given k_0 and $\gamma_{-1} = 0$, first order conditions for the FC equilibrium are

$$\lambda_t = u'(\mathrm{c}_t) + \gamma_t u''(\mathrm{c}_t) - \gamma_{t-1} u''(\mathrm{c}_t) \left(1 + \alpha \mathrm{k}_{t+1}^{\alpha-1} - \delta - \frac{\mathrm{g}_{t+1}}{\mathrm{k}_{t+1}}\right) \tag{57}$$

$$\lambda_t = z'(\mathrm{g}_t) + \gamma_{t-1} \frac{u'(\mathrm{c}_t)}{\mathrm{k}_t} \tag{58}$$

$$\lambda_t = \beta \lambda_{t+1}(1 + \alpha \mathrm{k}_{t+1}^{\alpha-1} - \delta) - \beta \gamma_t u'(\mathrm{c}_{t+1}) \left[\alpha(\alpha-1)\mathrm{k}_{t+1}^{\alpha-2} + \frac{\mathrm{g}_{t+1}}{\mathrm{k}_{t+1}^2}\right] \tag{59}$$

D.2 No Commitment

All future governments play $g_{t+j} = \tilde{g}^{NC}(\mathrm{k}_{t+j})$ for $j = 1, 2, ..,$ leading to continuation value $\tilde{V}^{NC}(\mathrm{k}_{t+1})$. The private sector follows the optimal consumption rule consistent with current and future government policies: $\mathrm{c}_t = \tilde{c}^{NC}(\mathrm{k}_t, \mathrm{g}_t; \tilde{g})$. The government problem, given state k is:

$$\max_{c,g} u(\mathrm{c}) + z(\mathrm{g}) + \beta \tilde{V}^{NC}(\mathrm{k}')$$

subject to

$$\mathrm{k}' = \mathrm{k}^\alpha + (1-\delta)\mathrm{k} - \mathrm{g} - \tilde{c}^{NC}(\mathrm{k}, \mathrm{g}; \tilde{g}) \tag{60}$$

and a NC equilibrium implies that the argmax of this maximization yields $\mathrm{g} = \tilde{g}^{NC}(\mathrm{k})$.

D.3 Limited-Time Commitment

Future governments play $\mathrm{g}_{t+j+1} = \tilde{g}^{LTC}(\mathrm{k}_{t+j}, \mathrm{g}_{t+j})$, leading to continuation value $\tilde{V}^{LTC}(\mathrm{k}_{t+1}, \mathrm{g}_{t+1})$. The private sector follows the optimal consumption rule consistent with current and future govern-

ment policies: $\mathrm{c}_t = \tilde{c}^{LTC}(\mathrm{k}_t, \mathrm{g}_t, \mathrm{g}_{t+1}; \tilde{g})$. Hence the continuation value can be expressed recursively as follows:

$$\tilde{V}^{LTC}(\mathrm{k}, \mathrm{g}) = u(\tilde{c}^{LTC}(\mathrm{k}, \mathrm{g}, \tilde{g}^{LTC}(\mathrm{k}, \mathrm{g}); \tilde{g})) + z(\mathrm{g}) + \beta \tilde{V}^{LTC}(\mathrm{k}', \tilde{g}^{LTC}(\mathrm{k}, \mathrm{g})) \tag{61}$$

with

$$\mathrm{k}' = \mathrm{k}^{\alpha} + (1-\delta)\mathrm{k} - \mathrm{g} - \tilde{c}^{LTC}(\mathrm{k}, \mathrm{g}, \mathrm{g}'; \tilde{g}) \tag{62}$$

The government problem, given state (k, g) is:

$$\max_{c,g'} u(\mathrm{c}) + z(\mathrm{g}) + \beta \tilde{V}^{LTC}(\mathrm{k}', \mathrm{g}')$$

subject to (62) and a LTC equilibrium implies $\mathrm{g}' = \tilde{g}^{LTC}(\mathrm{k}, \mathrm{g})$ as argmax.

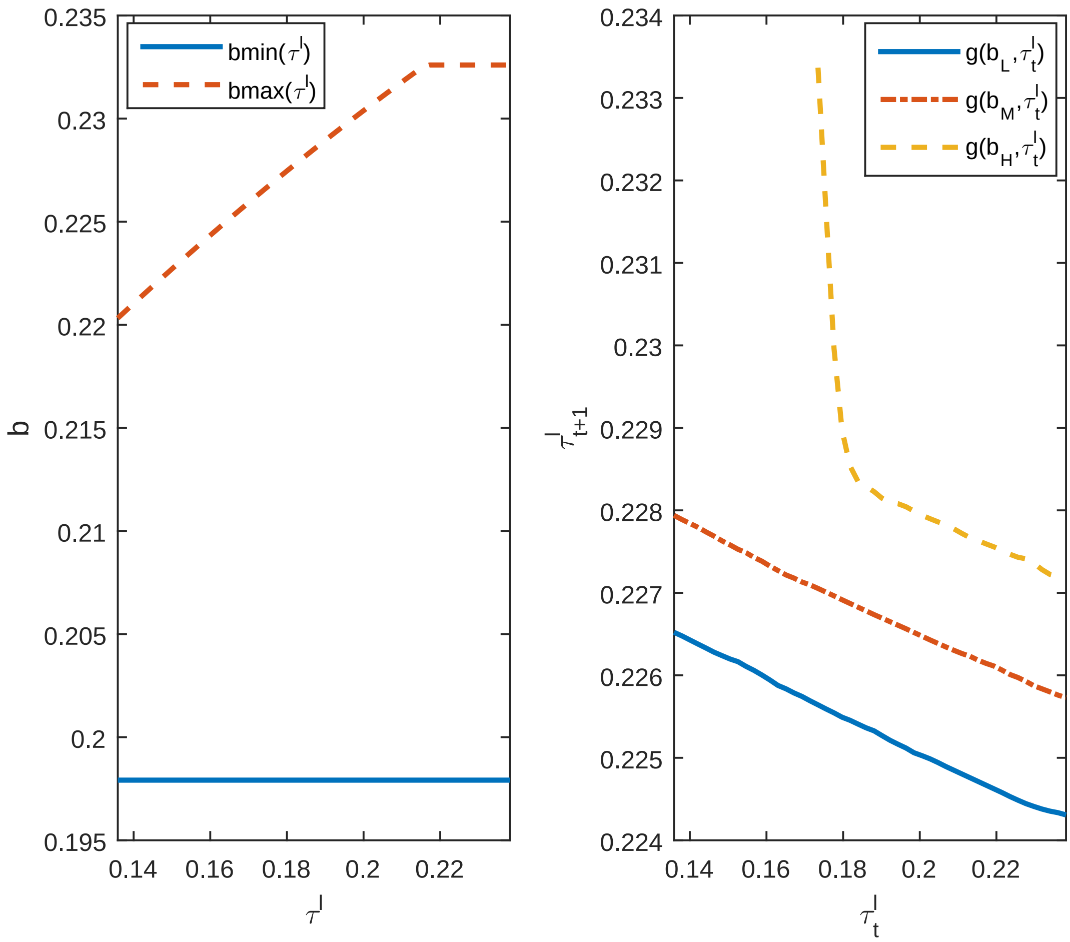
D.4 Algorithm for LTC

We briefly describe a global solution algorithm for LTC, based on projection methods. First, notice that thanks to separability between the two goods in utility, both in the consumer's saving problem and in the government's optimization, the capital stock and current government spending matter only through their joint effect on the amount of resources available for private consumption and investment, which we define as $\mathrm{a}_t \equiv \mathrm{k}_t^{1-\alpha} + (1-\delta)\mathrm{k}_t - \mathrm{g}_t$. Hence we can use this as a sufficient state variable to compute the decision rules. However, we need to use k and g independently to compute the continuation value function $\tilde{V}^{LTC}$. We proceed as follows.

1. Discretize the sets of a, k, g
2. Set a polynomial order S (in our case, $S = 3$) and guess a future policy function $\tilde{g}^{LTC} \approx P(\mathrm{a}; \phi_g) \equiv \sum_0^{S+1} \phi_{g,j} \mathrm{a}^j$
3. Solve the households' consumption-saving problem by iterating on the Euler equation (38) and approximating the consumption function $\tilde{c}^{LTC} \approx P(\mathrm{a}; \phi_c)$
4. Approximate the continuation value function $\tilde{V}^{LTC} \approx P(\mathrm{k}, \mathrm{g}; \phi_w)$ by iterating on (61)
5. Solve the maximization problem of government t on a grid for a. Approximate the associated decision rule with a polynomial with coefficients ϕ_g'
6. Update the guess for $\tilde{g}^{LTC}$ as follows: $\phi_g = \psi \phi_g' + (1-\psi)\phi_g$ with $\psi \in [0, 1]$ until convergence.

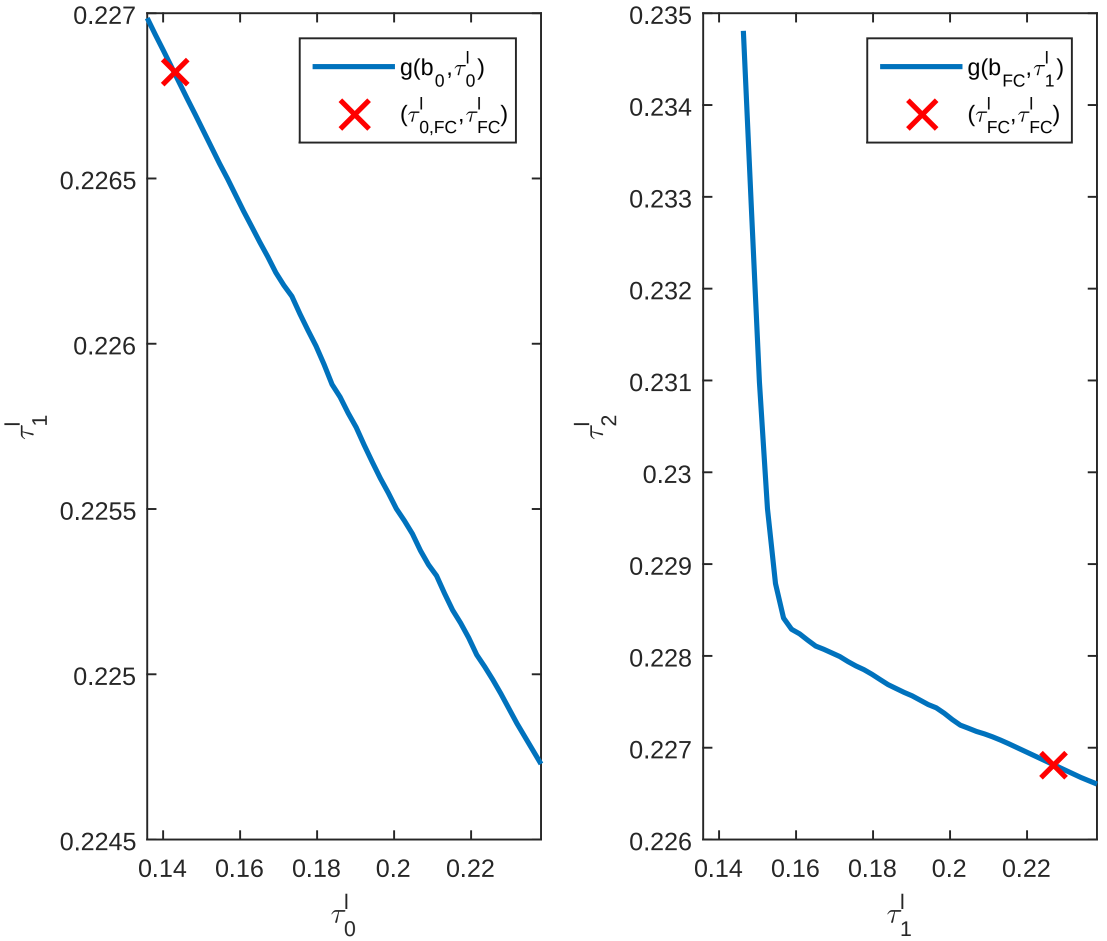
E Figures

Figure 1: Feasible set and LTC policy function in the LS economy



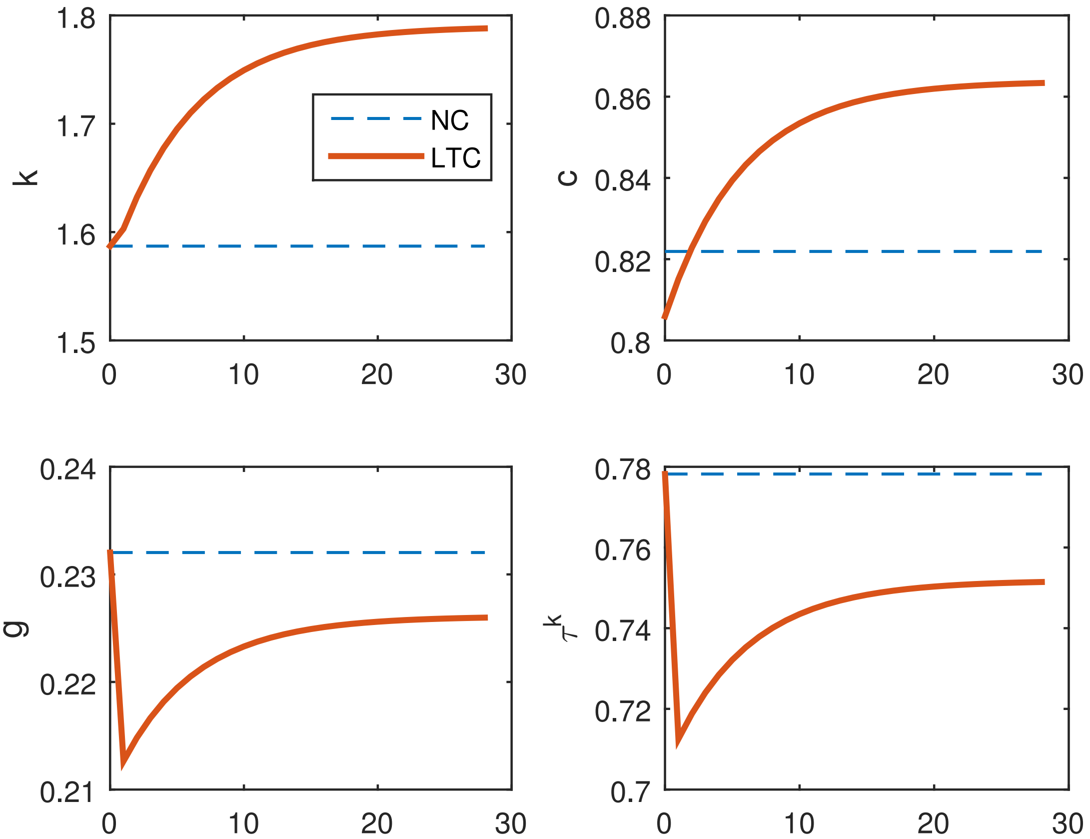
Left panel represents the feasible set X via upper and lower limits bmin(τ^l) *and bmax*(τ^l). *Right panel plots slices from the policy function* $\tau^l_{t+1} = g(\mathrm{b}_t, \tau^l_t)$ *at given values of* b_t, *where* τ^l_t *and* τ^l_{t+1} *are represented on the x- and y-axes respectively.* b_L, b_M, *and* b_H *represent low, medium, and high debt levels corresponding to 20% above the bottom of the grid for* b, *50% above, and 80% above.*

Figure 2: LTC and FC in the LS economy



Lines give LTC policy function $\tau^l_{t+1} = g(\mathrm{b}_t, \tau^l_t)$ at a given value of b_t, where τ^l_t and τ^l_{t+1} are represented on the x- and y-axes respectively. Crosses denote the t and $t+1$ values of the optimal FC plan. The left panel plots the $t = 0$ problems, showing the LTC plicy function for state b_0 and the time-0 and 1 FC taxes. The right panel plots the $t > 0$ problems, showing the LTC plicy function for state b_{FC} and the time-1 and 2 FC taxes, which are both equal to τ^l_{FC}.

Figure 3: A "constitutional reform": transition from NC to LTC



Transitional dynamics from NC steady-state (dashed blue) to LTC with one year of commitment (solid red). Top left panel: capital stock k_t*; top right: private consumption* c_t*; bottom left: government spending* g_t*; bottom right: tax rate* τ_t^k.

F Tables

Table 1: Parameter values

Parameter	Interpretation	Value
β	discount factor	0.96
D	public good utility	.5
α	capital share	.36
δ	depreciation	.08

Parameter values for the model in Section 6. We follow the yearly calibration in Klein et al. (2008).

Table 2: Steady-state comparison

Variable	FC	NC	LTC
y	1	.866	.905
k/y	1.735	1.344	1.452
c/y	.712	.696	.701
c/g	4.776	3.542	3.821
τ^k	.674	.778	.752
$welf.loss$	(0)	.096	.062

Steady-state results for the model in Section 6. We consider three versions of the economy (FC, NC and LTC) and report steady-state output, capital-output ratio, private consumption-output ratio, private consumption-public consumption ratio, tax rate and welfare loss, measured as the fraction of permanent consumption that would make the representative household indifferent between the economy considered and the FC economy.