# Growth, Business Cycles, and the Fear of Financial Crises

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**Abstract**

I study the ex ante effects of the fear of future financial crises. I show theoretically that this "crisis fear" has both negative growth and business cycle effects, and can overturn the conventional view of the tradeoffs of prudential policy. If agents anticipate the possibility of future crises, prudential policy can simultaneously increase growth and stabilise the economy, in contrast with common arguments that prudential policy should decrease growth. Productive experts fund investment by issuing debt to less productive households in a model featuring both business cycle shocks and endogenous growth. Crises are modelled through multiple equilibria: in some states the experts' net worth can be wiped out by a self-fulfilling fall in asset prices. I study the effects of allowing agents to anticipate such an event, by solving the model with a sunspot determining equilibrium selection. In a financial crisis, bankrupt experts sell their capital to less productive households, worsening the allocation of capital. Thus the possibility of future crises lowers the expected return on capital. This lowers asset prices and hence investment and growth today, even if experts are currently well enough capitalised to survive a crisis. The possibility of future crises also creates a state-dependent "financial crisis accelerator": shocks which push the economy closer to crisis lead to more severe financial accelerator effects than those that push the economy away from crisis.

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# 1 Introduction

Recent years have served as a painful reminder that modern economies are not safe from financial crises. While the eventual source of financial crises is often overlooked, looking forward there is a widespread perception that future crises are possible. A casual search for "next crisis" on Google News yields a long list of recent articles on the topic. Whether because changes in regulation in response to the last crisis were inadequate, or even laid the foundations for the next crisis,[1] or just because crises seem to happen every seven years,[2] there is no shortage of potential future crises. In this paper, I ask what the ex ante effects are of such "crisis fear".

I build a model featuring endogenous financial crises due to multiple equilibria, and study the general equilibrium effects of expectations of the possibility of future crises. The model is an extension of Brunnermeier & Sannikov's (2014a, henceforth BrS) continuous time model of an economy with a financial sector. Their model features multiplicity of equilibria in some regions of the state space: if leverage is high enough, a self-fulfilling fall in asset prices which bankrupts the financial sector is possible. Brunnermeier & Sannikov (2014b) point out this multiplicity, and the conditions under which it can arise. However, they do not model it formally in agents' expectations: ex ante, agents place zero probability on a crisis happening. My contribution is to treat crises as a sunspot event, allowing agents to understand the probability distribution over future crises. I then show that the fear of future crises has negative effects on both growth and business cycles, and implications for the tradeoffs of prudential policies.

The model features two classes of agents, productive "experts" and less productive "households". Experts borrow from households in order to buy capital to take advantage of their superior production technology. However, in some states of the world a self-fulfilling fall in asset prices is possible, since a fall in asset prices may bankrupt the expert sector, forcing them to liquidate their capital to unproductive households at fire-sale prices.

The first result is the effect of crisis fear on growth. During a crisis, the liquidation of capital to the less productive household sector reduces the average productivity of capital, and hence the possibility of future crises lowers the expected return on capital. This lowers asset prices and hence investment and growth today, even if experts are currently well-enough capitalised to survive a crisis. Thus I demonstrate that the potential future misallocation of capital across agents has implications

[1] "In the U.S. and Europe, the private sector's dependence on government support is fostering behaviors – excessive risk-taking, distortions in capital markets and maybe even inflationary pressures – that could lay the foundations for the next crisis." – WSJ (2012)

[2] "Financial crises come round every seven years on average. There was the stock market crash of 1987, the emerging market meltdown in the mid-1990s, the popping of the dotcom bubble in 2001 and the collapse of Lehman Brothers in 2008. If history is any guide, the next crisis should be coming along some time soon." – Guardian (2014)

for growth. The model features endogenous growth via an "$AK$" structure based on Romer (1986), so changes in the investment rate have permanent growth effects based on a a capital externality in production.[3]

The second result is the existence of a state-dependent "financial crisis accelerator": negative shocks which push the economy closer to the region where crises are possible reduce asset prices, which reduces expert net worth, bringing us even closer to the crisis region, reducing asset prices further in a vicious cycle. This operates on top of the traditional financial accelerator, making the overall size of the accelerator both state-dependent and asymmetric: positive shocks near the steady state have smaller effects than negative shocks. In terms of policy, these results suggest that there are both growth *and* stability benefits to dealing with financial crises, which are felt even in times when banks are well capitalised.

Thirdly, I show that while experts may deleverage in response to crisis fear in partial equilibrium, they may also actually take on *more* leverage in general equilibrium. This is because of general equilibrium price effects and limited liability. The fear of a crisis pushes down asset prices, which actually increases expected returns *conditional* on there not being a crisis in the near future. Since experts don't personally suffer all of the losses incurred during a crisis, the increase in this conditional return encourages experts to leverage up. On the other hand, experts are encouraged to deleverage because this reduces their borrowing costs by reducing the incidence of exogenous default costs. If the model features no exogenous default costs, then leverage is everywhere higher when agents fear crises. This adds an interesting interpretation to the link between leverage and crises: we typically think of high leverage causing crisis risk, but I show that the effect also works in the opposite direction, with crisis risk causing high leverage.

Finally, I show that several of BrS' results that are driven by exogenous shocks can be replicated using sunspots. In particular, their model features a bimodal distribution with serious but rare financial disasters driven by bad sequences of exogenous shocks. I show that several of their results hold if we instead consider rare crises driven by multiple equilibria. For example, they show that decreasing the volatility of exogenous shocks in their model increases the amount of endogenous volatility created in general equilibrium. I show that decreasing the volatility of exogenous shocks can increase the probability of experiencing a financial crisis, since lower volatility encourages experts to increase leverage, exposing themselves to crisis risk.

The model also highlights a crucial question: why would the financial sector expose itself to such

[3]If we considered a standard neoclassical growth model, the reduction in investment would instead slow down the transition to the balanced growth path and change the steady state level of capital. Modelling endogenous growth allows me to study the interaction between financial crises and long-run growth, and is computationally convenient because the equilibrium is linear in capital.

costly crisis risks? I show that in my model, in the absence of the endogenous-growth externality, an expert who is prudent and takes on low enough leverage to allow herself to survive a crisis would earn infinite value when the crisis hits. The reason for this is intuitive: it is wonderful to be the only expert in town. Prices are low and you operate a better technology than everyone else. In this case, crises cannot exist in equilibrium, since any expert would deviate and take on low leverage if others were taking on high enough leverage to put the economy at risk.

I show that, in the presence of my endogenous-growth externality, crises become possible in equilibrium. The structure I adopt has the productivity of an individual expert being dependent on the total capital managed by the aggregate expert sector. Thus, during a crisis, when other experts are liquidating their capital, your productivity is lowered. This removes the benefit of being the only surviving expert, and allows experts to coordinate on a high leverage, crisis-inducing equilibrium.[4]

My model has implications for the effects of prudential policy, which is often cast as a tradeoff between stability and growth. For example, it is often argued that leverage constraints reduce volatility, but reduce growth by reducing the ability of banks to intermediate capital on average (see, for example BIS, 2010). While this is still true in my model, the fear of crises introduces a counteracting effect, and well-designed prudential policies can simultaneously reduce volatility *and* increase growth.

The intuition for this result builds on the previously discussed growth and volatility results. The fear of future crises reduces growth, and thus prudential policies which reduce the probability (and hence fear) of a future crisis will tend to increase growth. In other words, leverage constraints could promote growth by making the system safer. This positive effect of policy on growth competes with the usual negative effects from reducing the ability of banks to raise funds. Thus whether prudential policy increases or decreases growth is ultimately a quantitative question of which effect dominates.

I consider a policy which forces experts to reduce leverage just enough to rule out crises at all times, which I call the "minimally active" leverage constraint. This policy is countercyclical in asset prices, requiring experts to hold lower leverage when asset prices are high. In a calibrated model, I show that this policy increases growth in equilibrium for a wide range of calibrations for the frequency of financial crises. Since experts' debt is reduced by the leverage constraint, the increase in growth must be financed by an increase in equity. In the model, experts are unable to issue equity, but equity increases because experts retain more earnings by consuming less, which can be interpreted as paying fewer dividends. They retain more earnings because eliminating crises increases the value

[4] As I discuss, there are empirical reasons to think the interpretation of it being wonderful to be the only expert in town is probably flawed. My assumption is meant to capture the idea that disruptions to financial markets during a crisis would make it hard to profit from low asset prices. An alternative explanation for experts coordinating on a high leverage equilibrium could be the expectation of receiving bailouts.

of internal net worth, since it is less likely to be lost during a crisis. If the frequency of crises is high enough, implementing this policy is welfare-improving regardless of the current state of the economy, while it is not welfare-improving in the same model if we ignore crisis risk.

Of course, whether such a policy is implementable in practice is an important question. The optimal degree of countercyclicality, or average level of the minimally active leverage constraint, requires knowledge of the structure of the economy that a policymaker might not possess. I show that the minimally active leverage constraint increases welfare, and investigate the effects of policies which are too tight or loose relative to this benchmark. Policy which is too tight can ultimately lead to the misallocation costs dominating, and hence lead to lower welfare. This suggests the existence of an "*inverse-U*" relationship between leverage policy and welfare: excessively tight leverage constraints will reduce welfare, excessively loose constraints will do nothing, and only intermediate policies can increase welfare. The degree of flexibility a policymaker has in trading these two effects off is a quantitative question, but the range of welfare improving policies is large in the baseline calibration.

Overall, the aim of the above policy discussion is to highlight that the common stability-vs-growth narrative may be overly simplistic. Even if hoping for an increase in growth from prudential policies might be too much to ask in a practical sense, the positive effects highlighted in this paper could mitigate the negative effects and ultimately reduce the costliness of such policies.

I also discuss bailout policies and market-based solutions. As in Diamond & Dybvig (1983), bailouts can completely rule out the bad equilibrium and hence incur no distortions, since they are never used in equilibrium. However, this is due to the simple structure of the model, and bailouts which are used in equilibrium may lead to distortions. Finally, I show how a simple market-based solution, offering experts insurance which pays off during a crisis, does not rule out crises. Indeed, the same forces which lead experts to be willing to take on high enough leverage to let themselves go bankrupt during a crisis are precisely those which lead them to adopt no insurance against the event.

## 2 Related Literature

My paper builds most on the ideas and framework of Brunnermeier & Sannikov (2014a). While they don't explicitly mention the possibility of crises in this paper, they discuss it in an international economics framework in Brunnermeier & Sannikov (2014b). In some states of the world, a second "bad equilibrium" exists in their model whereby experts can go bankrupt in a self-fullfilling crisis. While they discuss the existence of such an equilibrium, they do not allow agents to anticipate that a crisis might happen, and hence cannot discuss how the fear of a potential switch to the bad equilibrium affects behaviour ex ante.

Another recent paper which discusses financial crises in a general equilibrium framework is Gertler & Kiyotaki (2013). Their model features crises via the same mechanism (a fall in asset prices bankrupting banks), and additionally they discuss the effects of anticipated crises. My contribution relative to their paper is that I solve my model globally and nonlinearly, allowing me to study the state dependence of the effects of crisis fear, whereas they linearise around the non-stochastic steady state. Additionally I am able to study growth effects, whereas they assume a fixed stock of capital. Ennis & Keister (2003) also model the effect that the expectation of financial crises has on growth. They use an overlapping generations framework, and model financial crises in a way closer to to the original Diamond & Dybvig (1983) model. In their model, a crisis destroys a fraction of the capital stock, whereas in my model it worsens the allocation of capital across agents.

In an international context, Perri & Quadrini (2014) solve a two country model with financial crises due to multiple equilibria. Their focus is on explaining how the multiplicity implies that financial crises should be correlated across countries. Crises in their model are also anticipated events, with a known sunspot probability attached to them. Other papers also examine the link between financial frictions and multiple equilibria, and it is well known that financial frictions can lead to multiplicity. For example, Martin & Ventura (2014) and Kocherlakota (2009) present models with collateral constraints and multiple equilibria. The key idea is that higher asset prices relax collateral constraints, increasing the demand for assets and thus justifying the higher asset prices. Martin & Ventura (2014) also point out that the expectation that you might change equilibrium in the future affects today's equilibrium, which is a key theme of my paper.

The literature above and my paper could be viewed as a way to endogenise exogenous financial shocks, which have been shown to generate reasonable macroeconomic features in a recently emerging literature. Eggertson & Krugman (2012) and Jermann & Quadrini (2012) are two notable examples with models close to the representative agent framework, while Khan & Thomas (2013) and Guerrieri & Lorenzoni (2011) study heterogenous firm and consumer models respectively. Theoretically, one issue I am abstracting from is the ability of banks to issue equity. Admati & Hellwig (2013) emphasise that fears of the costs of leverage requirements could be overstated because banks can substitute equity for debt. In a Modigliani-Miller world this is exactly true, and in a world with frictions the costs of reducing debt depend on the relative size of the frictions in equity and debt issuance. My paper complements this idea by showing that even in the extreme case where banks cannot raise any equity the costs of leverage constraints might be overstated, since improvements to the stability of the system can encourage banks to retain earnings by paying less dividends.

Other papers provide evidence supporting or related to my model. Reinhart & Rogoff (2009) document that the historical average duration of recessions surrounding banking crises to be 1.9 years,

which they describe as being unusually long compared to normal recessions, which last on average less than a year. They find that the recoveries in unemployment tend to be even more protracted. Claessens & Kose (2013) find similar results, and additionally find that severe financial disruptions have slower recoveries than less severe.[5] My model can be interpreted as one rationalisation of this fact: if the fear of future crises rises (rationally or irrationally) following a financial crisis, then investment and growth will be unusually slow in the aftermath.

My model also revolves around the idea that financial crises are anticipated, in the sense that agents worry about future crises, understanding the states of the world in which they occur, and with what probabilities. This is consistent with evidence from financial markets that agents price in future "run risk" for individual financial institutions. Schroth, Suarez & Taylor (2014) present evidence from the asset-backed commercial paper (ABCP) crisis which started in July 2007, which has been widely been interpreted as a run due to the short term nature and widespread withdrawal of financing. They show empirically that spreads on ABCP forecast future runs, consistently with run risk being priced in to lending decisions. Additionally, the economic mechanism behind crises stressed in my paper is that a large fall in asset prices severely reduces the net worth of the financial sector. In historical data, Reinhart & Rogoff (2009) show that equity prices fall on average by 55.9% during banking crises, and that housing prices fall by an average of 35.5%. Additionally, Claessens & Kole (2013) show that asset prices also fall faster during financial crises than they do in normal recessions, which is supportive of their central role.[6]

Finally, my result that policy can simultaneously improve growth and reduce volatility relates to empirical work on the relationship between economic growth and volatility. Ramey & Ramey (1995) and Imbs (2007) present cross-country evidence that economies with lower volatility tend to have higher growth rates on average, which could be interpreted as broadly supportive of my proposed mechanism.

The rest of the paper is organised as follows. In the next section, I set up the model and describe some key features. In section 4, I solve a version of the model where agents place no weight in their expectations on crises happening, and present preliminary results. In section 5, I solve the full model where agents anticipate the possibility of future crises, and present the paper's main results. In section 6, I present policy results, and in section 7 I conclude.

[5] Some authors have argued that financial crises tend to have faster recoveries than normal recessions. Reinhart & Rogoff (2012) provide a summary of this argument, and evidence supporting their original work.

[6] The large fall in asset prices in the US during the recent crisis is well documented and broad-based, whether we look at equity or land aggregates, or more exotic financial securities. For example, the S&P500 index lost 45% of its value between September 2008 and March 2009, and over 50% from its peak in October 2007. The S&P/Case-Shiller 20-City Composite Home Price Index started falling much earlier, and dropped over 40% of its value between April 2006 and May 2009.

## 3 Model

The model is an extension of BrS' model to allow agents to expect crises, and the underlying framework is very similar. The derivations are thus very similar to the derivations in the original model, with the exception that I need to introduce a sunspot jump variable to allow agents to rationally take into account the possibility that the economy can experience a crisis. Additionally, to formalise the idea of endogenous growth I derive their linear production functions as a special case of Romer's (1986) growth model.

### 3.1 Technology

There are two types of agent, each with unit mass. *Households* are relatively inefficient at production compared to *experts*, but experts will be financially constrained and hence limited in their ability to accumulate capital. Production is carried out using capital using constant returns to scale production functions[7]: Each household has production function $\underline{y}_t = \underline{a}\underline{k}_t$, where under-bars are used throughout to denote household variables as opposed to expert variables. Each expert has production function $y_t = ak_t$, where $a > \underline{a}$. Productivity is constant over time for each class of agent, and the exogenous shock to the economy will instead be a capital quality shock. Capital is accumulated by converting the consumption good into capital. There are adjustment costs, so the price of capital, denoted by $q_t$, is not equal to unity. Household capital accumulation is given by:

$$d\underline{k}_t = \left(\Phi(\underline{\iota}_t) - \underline{\delta}\right)\underline{k}_t dt + \sigma \underline{k}_t dZ_t \tag{1}$$

Households' capital depreciates at the rate $\underline{\delta}$, and if they invest at rate $\underline{\iota}_t$ (where this is investment per unit of installed capital) they generate new capital at rate $\Phi(\underline{\iota}_t)$. Finally, their capital stock is subject to an aggregate Brownian shock, $dZ_t$, which has an effect proportional to their installed capital. This is the aggregate capital quality shock, whose variance is controlled by the parameter $\sigma$. Expert capital accumulates according to:

$$dk_t = \left(\Phi(\iota_t) - \delta\right)k_t dt + \sigma k_t dZ_t \tag{2}$$

Households have the same exposure to the shock as experts, and the same adjustment cost function, but have lower productivity and a potentially higher depreciation rate, $\underline{\delta} \geq \delta$. In the appendix I prove that these linear production functions can be viewed as the reduced form of a modification of Romer's (1986) endogenous growth model.

[7] There is no labour supply in the original BrS model. The linear production functions here should be considered linear *after* labour has been optimally chosen.

### 3.2 Sunspot & price process

In solving this kind of model it is typical at this point to conjecture that prices evolve as drift-diffusion process, with unknown drift and loading on the exogenous diffusion, $dZ_t$. However, given that this economy features a multiplicity of equilibria we can introduce a sunspot variable and conjecture that prices evolve as a function of this variable too. This obviously has to be verified in equilibrium. I guess that the capital price evolves according to:

$$dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t + (\underline{q} - q_t) df_t \tag{3}$$

Where $df_t$ is the change between $t$ and $t + dt$ in a counting variable which increases by one in that interval with probability $\rho_{e,t} dt$, and doesn't increase with probability $1 - \rho_{e,t} dt$. Thus the price evolves as a combination of time varying drift, diffusion from the exogenous capital quality shock, and a jump component from the sunspot. The final term is the sunspot term which says that the price could jump down from $q_t$ to $\underline{q}$ with some probability. This is what happens in a crisis. The cause of the fall in price will be the bankruptcy of all of the experts in the economy. Thus the jump in prices will also coincide with default and their joint consequences for the other variables of the economy.

The jump intensity, $\rho_{e,t}$ has both endogenous and exogenous components, and hence has to be determined in equilibrium. For example, in regions where banks are well enough capitalised to survive a crisis this will be endogenously zero. However, in regions where a crisis is possible, the modeller has freedom to choose how likely it is that a crisis occurs. As usual with models of multiple equilibria, there is nothing intrinsic in the model which tells us when we should switch equilibria. This means that I must look outside the model for equilibrium selection, which is why sunspots are required.

The crisis price, $\underline{q}$, is also endogenous and at this point undetermined. Intuitively, this is the price at which capital would trade if all experts go bankrupt and cease to intermediate capital. This is going to be lower than the current price, because this means that only inefficient households can purchase capital.

### 3.3 Markets

As well as the markets for consumption and for trading units of capital, there is a restricted set of financial markets. In particular, experts and households can only trade risky debt, and not state contingent claims (such as equity). Banks borrow from households at interest rate, $r_t$. In the case of default their net worth is reduced to zero, and households seize their capital less a proportional default cost.

### 3.4 Households

Households are risk neutral and allowed negative consumption, so their required expected return on any asset is always simply their subjective discount rate, $\rho_h$. Since experts may default in a crisis, they borrow at a rate above the risk free rate. In the appendix I show that the interest rate charged to an expert is:

$$r_t = \begin{cases} \rho_h & : \phi_t < \frac{1}{1-\frac{\underline{q}}{q_t}} \\ \rho_h + \rho_{e,t}\left(1 - (1-\chi)\frac{\underline{q}}{q_t}\frac{\phi_t}{\phi_t - 1}\right) & : \phi_t \geq \frac{1}{1-\frac{\underline{q}}{q_t}} \end{cases} \tag{4}$$

Where $\chi$ is the fraction of expert assets exogenously destroyed during default. $\phi_t \equiv \frac{q_t k_t^b}{n_t}$ is the leverage of a typical expert. The term in brackets is necessarily positive, leading to an interest rate spread, and it is possible to show that the interest rate is increasing in bank leverage. Note that the interest rate charged to any one expert depends optimally on that expert's own leverage, and not aggregate leverage.

Using the household capital evolution equation and conjectured price process we can calculate the household's return on holding capital. Ito's lemma with jumps gives us:[8]

$$d\underline{r}_t^k = \left(\frac{\underline{a} - \underline{\iota}_t}{q_t} + \Phi(\underline{\iota}_t) - \underline{\delta} + \mu_t^q + \sigma\sigma_t^q\right)dt + (\sigma + \sigma_t^q)\,dZ_t + \frac{\underline{q} - q_t}{q_t}df_t \tag{5}$$

Note that the realised return on capital could involve a non infinitesimal loss in the case of a jump. The expected return will remain infinitesimal because this only happens with probability of order $dt$: $E_t df_t = \rho_{e,t}dt$. The expected return for a household is:

$$\mathrm{E}d\underline{r}_t^k = \left(\frac{\underline{a} - \underline{\iota}_t}{q_t} + \Phi(\underline{\iota}_t) - \underline{\delta} + \mu_t^q + \sigma\sigma_t^q + \frac{\underline{q} - q_t}{q_t}\rho_{e,t}\right)dt \tag{6}$$

In continuous time the investment decision is a static problem, and we can determine the optimal investment rate as the rate that maximises the above return (or equivalently the expected return). Differentiation with respect to $\underline{\iota}_t$ yields:

$$\Phi'(\underline{\iota}_t) = \frac{1}{q_t} \tag{7}$$

Thus the optimal investment rate depends only on the current price of capital, and from now on we can think of $\underline{\iota}_t$ as implicitly being defined by $q_t$. Households are not allowed to short the capital stock so, given the assumption of risk neutrality, either they hold zero capital, or are indifferent about their capital holdings, or want to hold infinite capital (which is ruled out by market clearing). Thus we can summarise the household's capital optimality conditions as:

$$\frac{\underline{a} - \underline{\iota}_t}{q_t} + \Phi(\underline{\iota}_t) - \underline{\delta} + \mu_t^q + \sigma\sigma_t^q + \frac{\underline{q} - q_t}{q_t}\rho_{e,t} \leq \rho_h \tag{8}$$

[8] The return is composed of a dividend $\underline{a}$ plus a capital gains term. Capital gains are computed as $\frac{d(q_t\underline{k}_t)}{q_t\underline{k}_t}$ which includes the capital gain from price changes and from changes in the capital stock itself, due to either the shock or investment and depreciation.

Which holds with equality if the household holds capital. Define $\psi_t \equiv k_t/K_t$ as the share of the total capital stock owned by experts (where $k_t$ is the integral over the identical holdings of the unit mass of experts). Then the above inequality is binding in equilibrium if and only if $\psi_t < 1$.

Finally, it is convenient at this point to ask what the price of capital would have to be if the household was to hold all of the capital stock forever (i.e. if the household was the only agent in this economy). In this case we can use the household's capital FOC, (8), to give the capital price. Guessing that in this case the price of capital is constant ($\mu_t^q = \sigma_t^q = 0$) gives:

$$\frac{\underline{a} - \underline{\iota}_t}{q_h} + \Phi(\iota(q_h)) - \underline{\delta} = \rho_h \Rightarrow q_h = \frac{\underline{a} - \iota(q_h)}{\rho_h + \underline{\delta} + \Phi(\iota(q_h))} \tag{9}$$

We see that our guess that $\mu_t^q = \sigma_t^q = 0$ is confirmed since there are no time varying elements in the above equation.

### 3.5 Experts

Experts are also risk neutral, and have subjective discount rate $\rho_b$. Experts, unlike households, must have non-negative consumption. This means that they can become financially constrained, because if they are lacking in net worth they will only be able to expand capital holdings by issuing risk free debt, which comes at the cost of magnifying risk. Denote an expert's net worth by $n_t$, where this is the market-to-market book value of her assets minus liabilities. I define the current maximised value of her utility by $\theta_t n_t$. Thus $\theta_t$ is an expert's value per unit of net worth, which I call experts' "marginal value", and which will be a function of the aggregate state in equilibrium. Conjecture that marginal value follows:

$$d\theta_t = \mu_t^\theta \theta_t dt + \sigma_t^\theta \theta_t dZ_t + df_t \left(\underline{\theta}_t - \theta_t\right) \tag{10}$$

Where $\underline{\theta}_t$ is the value of an expert's marginal value following a crash, which is to be determined. An expert earns the following return on capital:

$$dr_t^k = \left(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right) dt + (\sigma + \sigma_t^q)\, dZ_t + \frac{\underline{q} - q_t}{q_t} df_t \tag{11}$$

This return is maximised by the same choice for investment as households: $\Phi'(\iota_t) = 1/q_t$. They pay an interest rate which depends on how much leverage they take on, as in (4). I derive the solution to the expert's problem in the appendix. A very important issue is what happens to the individual expert during a crisis. Remember that the crisis is an aggregate event, and the expert does not think that she has any power to affect whether it happens or not. This is because a crisis is a fall in asset prices, which individual agents take as given. But an expert does have the power to control whether or not she personally goes bankrupt during a crisis. To see this, note that if the price of

capital instantaneously falls from $q_t$ to $\underline{q}$, an expert's net worth falls to:

$$\underline{n}_t = \max\left\{n_t - q_t k_t + \underline{q} k_t, 0\right\} = \max\left\{n_t\left(1 - \phi_t\left(1 - \frac{\underline{q}}{q_t}\right)\right), 0\right\} \tag{12}$$

The expert will go bankrupt during a crisis if the term inside the max operator is less than zero, leaving $\underline{n}_t = 0$, and will survive the crisis if $\underline{n}_t \geq 0$. Note that while the occurrence of a crisis is a random event, it is completely deterministic whether an expert survives the crisis. If prices fall in a crisis then $1 - \underline{q}/q_t > 0$, meaning that $\underline{n}_t$ decreases in leverage, and an expert can be certain to survive the crisis by choosing low enough leverage. In the extreme, an expert could choose to buy no capital ($\phi_t = 0$) and be certain to always survive a crisis, since then $\underline{n}_t = n_t > 0$.

This introduces a kink into the expert's value function, because there is a threshold leverage choice $\hat{\phi}_t = q_t/(\underline{q} - q_t)$ above which the expert goes bankrupt in a crisis, and below which she does not. Thus a key element of equilibrium will be which region the expert optimally chooses: a crisis cannot be an equilibrium if the existence of a crisis causes all of the experts to deleverage to avoid it. Understanding the conditions under which experts do and do not expose themselves to this crisis risk is clearly an important question. However, the focus of this paper is on understanding the general equilibrium effects of such crisis risk, and as such I make assumptions to ensure that if all other experts are choosing high enough leverage to make a crisis possible then you are also happy to take on that high level of leverage.

In particular, the appendix details the endogenous growth structure behind the reduced-form linear production technologies used in the main text. These use an aggregate capital externality based on Romer (1986), whereby the productivity of any individual expert depends on the total capital being intermediated by the entire *expert sector*. In this case, if all other experts take on enough leverage to expose themselves to a crisis, you have a strong incentive to as well. This is because during a crisis the rest of the experts will have to shed all of their capital, reducing your own productivity during a crisis via the capital externality. In the appendix I prove that this leads to experts optimally maintain high enough leverage to expose themselves to a crisis, if all other experts are doing so.[9]

I relegate the derivation of the expert's problem to the appendix, and present the key results

[9] In the appendix I provide a thorough discussion of the conditions under which an expert would allow herself to go bankrupt during a crisis. My technological assumption is a stand in for the idea that disruption in financial markets during a crisis makes it hard to take advantage of the high expected returns that come with temporarily low asset prices. Alternatively, the expectation of bailouts could explain why institutions expose themselves to such risk. The question then becomes why individuals do not reduce their exposure to the crisis. In the words of Citygroup chief executive Chuck Prince, "When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing." (FT, 2007)

here. The expert's leverage first order condition gives:

$$\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q - \rho_h = -\sigma_t^\theta\left(\sigma + \sigma_t^q\right) + \rho_{e,t}\left(1 - (1-\chi)\frac{\underline{q}}{q_t}\right) \tag{13}$$

This is identical to BrS' original FOC, with the addition of the final term. The FOC can be interpreted as a simple marginal benefit / marginal cost comparison. On the left hand side is the marginal benefit of increasing leverage, which is the expected excess return on capital over the risk free rate. On the right hand side is the marginal cost of increasing leverage. The first term is common with the original BrS model, and is the increase in risk the expert faces by increasing leverage. The financial friction makes the expert effectively risk averse, and since she has to finance risky capital using less than fully-contingent debt her risk is magnified as she leverages up. The final term is the marginal cost of increasing leverage related to crises. By leveraging up the expert splits her collateral more thinly across her creditors, leading to a higher required interest rate as there is less security per unit lent. The first order condition for consumption gives:

$$\theta_t \geq 1 \tag{14}$$

with equality if $dc_t > 0$. The marginal utility gained from consuming today is always one due to the risk neutrality assumption. As in BrS, the expert thus consumes nothing if the marginal value of retaining earnings exceeds one. If the value of retaining earnings ever falls to one the expert is indifferent about consuming and holding capital, placing a lower bound of one on $\theta_t$. At the optimum, and when $dc_t = 0$, evaluating the value function gives:

$$\mu_t^\theta = \rho_b - \rho_h \tag{15}$$

This gives us our solution for $\mu_t^\theta$, and solving the expert's problem now just requires finding a solution for $\sigma_t^\theta$ at every point in the aggregate state space. One important thing to note is that neither of the optimality conditions pins down an exact value for optimal leverage. The expert is in fact indifferent about her choice of leverage as long as (13) holds. This is a product of the risk neutrality assumption, and surprisingly even holds when we add crisis risk.

### 3.6 Market clearing

In equilibrium prices adjust to clear the consumption, capital and bond markets. Consumption market clearing requires that expert and household consumption and investment flows equal the flow of production. The risk neutrality of households ensures that this market clears. Capital market clearing requires that expert and household capital demand sums to the supply of installed capital:

$$k_t + \underline{k}_t = K_t \tag{16}$$

Where $K_t$ is the current stock of capital, which is an aggregate state variable. In equilibrium, the price of capital adjusts so that the sum of expert and household capital demand equals the existing total installed capital stock. In practice, given the linearity of both sets of agents' policy functions, this means adjusting the price to ensure that they are either indifferent about their capital holdings, or don't want to hold any at all.

Experts will always hold capital in equilibrium, since they are more productive than households. Households may not find it profitable to do so depending on the current return. There are thus two regions of the state space, corresponding to whether or not the household holds capital. If the household doesn't hold capital then we determine the price of capital using the expert's leverage first order condition, (13), and need to check that (8) holds with inequality. That is, in this region:

$$\frac{a-\iota_t}{q_t}+\Phi(\iota_t)-\delta+\mu_t^q+\sigma\sigma_t^q-\rho_h=-\sigma_t^\theta\left(\sigma+\sigma_t^q\right)+\rho_{e,t}\left(1-(1-\chi)\frac{\underline{q}}{q_t}\right)$$

And:

$$\frac{\underline{a}-\underline{\iota}_t}{q_t}+\Phi(\underline{\iota}_t)-\underline{\delta}+\mu_t^q+\sigma\sigma_t^q+\frac{\underline{q}-q_t}{q_t}\rho_{e,t}<\rho_h$$

Since the household is not holding capital, we only need to adjust the price of capital to make sure the expert is indifferent about her leverage choice. In the region where the household does hold capital, both agents' FOCs hold with equality, so we can set (8) to bind and subtract it from (13) to give:

$$\frac{a-\underline{a}}{q_t}+\underline{\delta}-\delta-\rho_{e,t}\chi\frac{\underline{q}}{q_t}+\sigma_t^\theta\left(\sigma+\sigma_t^q\right)=0 \tag{17}$$

This is the counterpart to BrS' equation (17) when we allow for anticipated crises. This equation is important because it tells us what has to be true to make experts and households simultaneously happy to hold capital. Intuitively, we have to somehow make both experts and households indifferent about their capital holdings, even though experts are more productive and would hence tend to be happier to intermediate. Notice that in the absence of the risk or jump terms this would be impossible since $a>\underline{a}$ and $\underline{\delta}>\delta$, and no $q_t$ could make the above equation hold.

In the original BrS model without crises the capital market is actually cleared by adjusting the level of endogenous risk. To make both sets of agents happy to hold capital the experts' productivity advantage is offset by the fact that they dislike risk, while the household does not. This is where the actual value of expert leverage is determined. Remember it is not pinned down by individual optimisation, since they are individually indifferent about their leverage levels. The level of leverage is determined in general equilibrium, as the level which creates enough risk to satisfy the above equation. Remember that increasing leverage increases risk, by increasing the experts' exposure to risky assets while funding them with risk free debt.

In the model with anticipated crises, there is also the jump term to consider. If we are in the crisis-prone region and $\chi>0$ then the jump term tends to reduce leverage, all else equal, because

it pushes up expert borrowing rates and increases the marginal cost of leverage. In order to reduce the marginal cost to restore indifference we need to reduce the amount of endogenous risk, which is achieved by lower leverage in equilibrium as discussed above.

The other state variable is total bank net worth, $N_t$. The capital quality shock is i.i.d and therefore there is no need to include it's value as a state. The sunspot also doesn't introduce a new state variable conditional on the current state, since it is simply a flow probability of moving between equilibria. Thus we can completely describe the equilibrium of the economy on the state space $(N_t, K_t)$.

### 3.7 State space representation

I noted that the state variables are $(N_t, K_t)$. BrS show that the equilibrium of the economy scales linearly in $K_t$ if we use bank net worth as a proportion of total net worth as a state instead of $N_t$:

$$\eta_t \equiv \frac{N_t}{q_t K_t} \tag{18}$$

In other words, we use $(\eta_t, K_t)$ as a state. Most variables, such as $q_t$ and $\phi_t$, will only depend on $\eta_t$ in equilibrium. Others, such as total consumption, output and capital demand, will depend on $\eta_t$, but also scale linearly in $K_t$. Conjecture the following law of motion for $\eta_t$:

$$d\eta_t = \mu_t^\eta \eta_t dt + \sigma_t^\eta \eta_t dZ_t - \eta_t df_t \tag{19}$$

The drift and volatility terms are to be determined along with the drifts and volatilities for $q_t$ and $\theta_t$. The jump term says that in a crisis the value of $\eta_t$ jumps down to zero. This is because in a crisis $N_t$ jumps to zero, and hence so does $N_t/(q_t K_t)$. The variables $q_t$, $\theta_t$ and $\psi_t$ are functions only of $\eta_t$ in equilibrium. Given $\psi_t$ and $\eta_t$ it is easy to calculate leverage as $\phi_t = \psi_t/\eta_t$. Using Ito's lemma we can solve for all of the unknown drifts and volatilities as functions of the parameters and the unknown functions above. I relegate the derivations to the appendix.

### 3.8 Crisis equilibrium

At this point we can construct the crisis equilibrium. Suppose that at some time $t$ experts have leverage $\phi_t$, net worth $N_t$, and face a price of capital of $q_t$. The multiplicity I investigate is whether a fall in price from $q_t$ to $\underline{q}$ is enough to bankrupt the experts. The fall in price causes net worth to fall from $N_t$ to:

$$\underline{N}_t = N_t - q_t k_t + \underline{q} k_t = N_t \left(1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right)\right) \tag{20}$$

If $\underline{N}_t < 0$ the experts go bankrupt. Given knowledge of $\underline{q}$, a crisis is thus possible at the current state if:

$$1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right) < 0 \tag{21}$$

The question is now whether the bankruptcy of experts can *justify* why the price fell to $\underline{q}$. This is true if the equilibrium price post bankruptcy is $\underline{q}$. Given that the price is a function of the state of the economy, $\underline{q}$ is an equilibrium object: it is the price that capital trades at once experts go bankrupt. Thus calculating this price requires specifying what happens in the economy after a crisis.

At the moment a crisis hits expert net worth drops to zero, as they do not have enough to pay off their creditors. With zero net worth experts have no money to purchase capital, and they cannot leverage off zero net worth. Hence households have to hold the whole capital stock in equilibrium ($\psi_t = 0$) immediately following a crisis. However, this is not enough information to price capital, since even the household's first order conditions are forward looking, and so the crisis price of capital will depend on the expectation of future prices.

However, note that experts' policy functions are linear in net worth. This means that once their net worth drops to zero, it must remain there indefinitely. Intuitively, with no net worth they cannot invest, and hence cannot generate any new net worth. This means that, in the absence of any intervention, a financial crisis would be permanent in this model. This makes it easy to price capital: the price when households hold the whole capital stock forever can be simply solved from (9). For simplicity, this is the assumption I maintain in the baseline model.

To facilitate non-permanent crisis, I consider the following extension. In order to spur the recovery of the experts, I can introduce an exogenous equity injection to restore them to positive net worth. This can be thought of as originating from either households or from a government sector. Specifically, I assume that following a crisis, the expert sector will receive an equity injection with probability $\rho_r dt$ in any interval $dt$. This injection is sufficient to restore $\eta$ to some $\hat{\eta}$, and once it is given no further equity injections are given until another crisis occurs. Once the equity injection is given, capital is priced via the normal equilibrium and hence the price jumps up to $q(\hat{\eta})$. Until this happens only households are holding capital, and hence we can price capital using the household's capital first order condition. The price will be constant at $\underline{q}$ until the experts are recapitalised. Taking this into account, the return a household earns on capital during a crisis is:

$$d\underline{r}_t^k = \left(\frac{\underline{a} - \underline{\iota}_t}{\underline{q}} + \Phi(\underline{\iota}_t) - \underline{\delta}\right) dt + \sigma dZ_t + \frac{q(\hat{\eta}) - \underline{q}}{\underline{q}} dg_t \tag{22}$$

The drift and diffusion terms give the return if we remain in a crisis. Note that during a crisis the price is constant at $\underline{q}$, so there are no price appreciation or volatility terms. The last term gives the capital gain the household makes in the even that experts are recapitalised, which is the jump event $dg_t$. In equilibrium the expected return equals the household's subjective discount rate:

$$\rho_h = \frac{\underline{a} - \underline{\iota}_t}{\underline{q}} + \Phi(\underline{\iota}_t) - \underline{\delta} + \frac{q(\hat{\eta}) - \underline{q}}{\underline{q}} \rho_r \tag{23}$$

This gives one equation to determine $\underline{q}$ given $q(\hat{\eta})$. Notice that as $\rho_r \to 0$ the price of capital approaches the price of capital if households are expected to hold all of the capital stock forever.

Once $\hat{\eta}$ and $\rho_r$ are chosen, and knowing the equilibrium price function in normal times, $q(\eta)$, we thus have enough information to calculate the price of capital during a crisis. I solve a version of the model with $\rho_r > 0$ in the appendix, and restrict myself to permanent crises ($\rho_r = 0$) for the baseline calibration.

The fundamental cause of crises in this model is the *misallocation of capital.* A crisis causes experts to go bankrupt, which pushes all of the capital stock into the hands of inefficient households. Since these households are inefficient and produce less from the capital stock the price of capital falls to reflect this. This model is thus fundamentally a model of multiplicity due to endogenous misallocation of capital, and a crisis manifests itself as a drop in measured total factor productivity (TFP). An important question is whether modelling the real effects of crises as a drop in measured TFP is empirically correct, a question I tackle in depth in my second chapter.

### 3.9 Selecting equilibria

In a region of the state space where a crisis is possible, between $t$ and $t + dt$ the economy carries on as usual with probability $1 - \rho_{e,t}dt$, and experiences a crisis with probability $\rho_{e,t}dt$. The intensity $\rho_{e,t}$ is a variable that the modeller gets to choose, and intuitively controls the probability that agents coordinate on the belief that a self fulfilling fall in asset prices will happen. A simple assumption would be to set $\rho_e$ to a constant value, which would mean the probability of experiencing a crisis is constant (whenever a crisis is possible). This leads to the following form for $\rho_{e,t}$:

$$\rho_{e,t} = \begin{cases} \rho_e & : \underline{N}_t < 0 \\ 0 & : \underline{N}_t \geq 0 \end{cases} \tag{24}$$

So far I have only discussed switching from the "normal" equilibrium to the crisis equilibrium, but is it possible to switch back? The answer is no, as once we enter a crisis expert capital holdings fall to zero, and it is hence not possible for changes in capital prices to push expert net worth around, which was the mechanism for our jumps. Instead, we return to the normal equilibrium once experts' net worth is restored via an equity injection.

### 3.10 Equilibrium

This section is the equivalent to BrS' Proposition II.4 in my model. We solve for the unknown functions $q$, $\theta$ on state space $\eta \in [0, \eta^*]$. All other variables are implicitly defined by $(q, \theta, \eta)$. At any point in the state space where $\underline{N}(\eta) < 0$ the economy may experience a crisis, and does so at the endogenous rate $\rho_{e,t}$ which is solved for along with the other equilibrium variables. $\eta^*$ is an upper bound on $\eta$ because at this point returns are so low that experts consume their net worth. There

are five boundary conditions:

$$q(0) = q_h \qquad \theta(\eta^*) = 1 \qquad q'(\eta^*) = 0 \qquad \theta'(\eta^*) = 0 \qquad \lim_{\eta \to 0} \theta(\eta) = \infty \tag{25}$$

Given the current state, $\eta$, and $(q(\eta), q'(\eta), \theta(\eta), \theta'(\eta))$ we can calculate $(q''(\eta), \theta''(\eta))$ using the following procedure.

1. Find $\psi$ such that

$$\frac{a - \underline{a}}{q_t} + \underline{\delta} - \delta - \rho_{e,t}\chi \frac{\underline{q}}{q_t} + \sigma_t^\theta \left(\sigma + \sigma_t^q\right) = 0 \tag{26}$$

where:

$$\sigma_t^\eta \eta = \frac{(\psi - \eta)\sigma}{1 - (\psi - \eta)q'(\eta)/q(\eta)} \qquad \sigma_t^\theta = \frac{\theta'(\eta_t)\sigma_t^\eta \eta_t}{\theta_t} \qquad \sigma_t^q = \frac{q'(\eta_t)\sigma_t^\eta \eta_t}{q_t} \tag{27}$$

and

$$\rho_{e,t} = \begin{cases} \rho_e & : 1 - \frac{\psi}{\eta}\left(1 - \frac{\underline{q}}{q_t}\right) < 0 \\ 0 & : 1 - \frac{\psi}{\eta}\left(1 - \frac{\underline{q}}{q_t}\right) \geq 0 \end{cases} \tag{28}$$

If $\psi > 1$ set $\psi = 1$ and recalculate (27).

2. Compute

$$\mu_t^\eta = -\sigma_t^\eta(\sigma + \sigma_t^q + \sigma_t^\theta) + \rho_{e,t}(1 - \chi)\frac{\underline{q}}{q_t} + \frac{a - \iota_t}{q_t} + (1 - \psi_t)(\underline{\delta} - \delta) \tag{29}$$

$$\mu_t^q = \rho_h - \frac{a - \iota_t}{q_t} - \Phi(\iota_t) + \delta - \sigma\sigma_t^q - \sigma_t^\theta\left(\sigma + \sigma_t^q\right) + \rho_{e,t}\left(1 - (1 - \chi)\frac{\underline{q}}{q_t}\right) \tag{30}$$

$$\mu_t^\theta = \rho_b - \rho_h \tag{31}$$

$$q''(\eta) = \frac{2\left[\mu_t^q q(\eta) - q'(\eta_t)\mu_t^\eta \eta\right]}{\left(\sigma_t^\eta\right)^2 \eta^2} \qquad \theta''(\eta) = \frac{2\left[\mu_t^\theta \theta(\eta) - \theta'(\eta_t)\mu_t^\eta \eta\right]}{\left(\sigma_t^\eta\right)^2 \eta^2}$$

Finally, $\hat{\eta}$ is a given parameter, and the crisis price, $\underline{q}$, solves:

$$\rho_h = \frac{\underline{a} - \underline{\iota}_t}{\underline{q}} + \Phi(\underline{\iota}_t) - \underline{\delta} + \frac{q(\hat{\eta}) - \underline{q}}{\underline{q}}\rho_r \tag{32}$$

### 3.11 Looking through the equations in partial equilibrium

To begin understanding the effect that anticipated crises have on equilibrium, it is helpful to look through the equations of the model, as summarised above. This allows us to trace where crises enter the equilibrium and gain some intuition as to the effects. Notice that the probability of a crisis, $\rho_{e,t}$, enters via three equations: (26), (29) and (30).

### 3.11.1 Equation 26: Leverage

Starting with equation (26), the following result is useful:

**Proposition 1.** *Suppose we are at a given state $\eta$ in the interior of the crisis region, and where both experts and households intermediate capital. Then for given $(q(\eta), q'(\eta), \theta(\eta), \theta'(\eta))$ an increase in the probability $\rho_e$ of selecting the crisis equilibrium reduces expert leverage if and only if $\chi > 0$.*

Note that the reason we condition on $(q(\eta), q'(\eta), \theta(\eta), \theta'(\eta))$ in the proof is that this parallels how we actually solve for leverage in the numerical solution. The proof is simple and relegated to the appendix, but the intuition is more subtle and I discuss it here. This is not a true partial equilibrium experiment, in the sense that we study a single agent's actions when we take prices as exogenous. To see this note that I have taken expert value, $\theta$, as given too.

Remember that we form equation (26) by subtracting the household's capital first-order condition (if it is binding) from the expert's leverage first order condition. Thus (26) holds if both agents are holding capital, and is necessary for them both to be indifferent about their capital holdings. $\rho_{e,t}$ only enters equation (26) if $\chi > 0$, and does not if $\chi = 0$. This means that in the absence of exogenous default costs ($\chi = 0$) crisis risk has no immediate impact on who prefers to hold capital.

To see this, note that increasing the probability of selecting a crisis affects the optimality conditions for *both* experts and households. For households, it increases the probability that they suffer a large capital loss on the capital in a crisis, reducing their expected return on capital, as in (8). For experts the story is slightly different. While an increase in the probability of a crisis does reduce their expected return, they do not care directly about this. This is because they go bankrupt during a crisis, and hence place no weight on this loss in their optimisation. Instead, experts only care about the probability of a crisis because it affects the spread they pay on their debt. When experts consider taking on an extra unit of leverage they understand that they will have to repay one extra unit at the current interest rate $r(\phi)$, and will also face a marginal increase in their interest payments $r'(\phi)$ because their collateral becomes more thinly spread across their debtors. An increase in the probability of a crisis increases both the current interest rate and the marginal increase.

Interestingly, when $\chi = 0$ these effects are of exactly the same magnitude, and crisis risk does not hurt one group more than the other. Thus when we subtract the two first order conditions from each other, no terms involving $\rho_{e,t}$ remain. However, when $\chi > 0$ this is not true, and a term involving $\rho_{e,t}$ does remain. This term causes us to reduce leverage in response to crisis risk. Intuitively, this is because when we add exogenous default costs the interest rate the expert pays increases faster as she increases leverage, encouraging her to deleverage.

In sum, if exogenous default costs are positive, increasing the probability of a crisis harms both the experts and the household, but harms the expert more on the margin. Given this, to restore both

agents to indifference we will need to help the expert, which is achieved by reducing equilibrium leverage. This helps the expert in two ways. Firstly, reducing equilibrium leverage reduces the amount of endogenous risk, as discussed in Section 3.6, which helps experts since they are effectively risk averse. Secondly, reducing leverage reduces the amount by which the experts are undercapitalised in a crises, which reduces the probability of selecting the crisis equilibrium as per (28).

Thus I have established that in partial equilibrium anticipation of crises reduces expert leverage if and only if $\chi > 0$, which pushes capital into the hands of inefficient households. However, we will see in the numerical section that this partial equilibrium result is overturned by general equilibrium forces in some areas of the state space, meaning that leverage could even increase in response to crisis risk.

#### 3.11.2 Equation 30: Capital gains

Equation (30) allows us to compute the drift in the capital price, $\mu_t^q$ once we have computed the volatilities. Since $1 - (1-\chi)\underline{q}/q_t > 0$, we see that this equation gives us a higher $\mu_t^q$ whenever $\rho_{e,t}$ is higher, all else constant.

The intuition for why this is the case is relatively simple. Equation (30) is the expert's leverage first order condition, rearranged to solve for $\mu_t^q$. Increasing $\rho_{e,t}$ increases the expert's marginal cost of leverage by increasing the interest rate the expert pays on debt. In order to return the expert to indifference about her leverage choice we must increase the marginal benefit of leverage, which is done by increasing the expected return to capital. Since we are (for the purposes of this discussion) taking all else as given, this is achieved by increasing $\mu_t^q$ which increases capital gains.

This reasoning leads us to expect that we should expect that expected returns *conditional on there not being a crisis* should be higher when agents attach a high probability to there being a crisis in the near future. We can see this by rearranging (30) and using the definition of $dr_t^k$:

$$\mathrm{E}_t\left[dr_t^k | df_t = 0\right] = \rho_h - \sigma_t^\theta\left(\sigma + \sigma_t^q\right) + \rho_{e,t}\left(1 - (1-\chi)\frac{\underline{q}}{q_t}\right) \tag{33}$$

While this is true for the expected return conditional on there being no crisis, it is less so for the overall expected return. Nonetheless, the conditional expected return is interesting because it tells us how the economy behaves in the scenario that we don't experience a crisis.

#### 3.11.3 Equation 29: Expert net worth

The final equation $\rho_{e,t}$ appears in is the drift of the aggregate state, $\mu_t^\eta$. The aggregate state is expert net worth as a fraction of the value of the capital stock, and tends to grow when expert net worth grows. Like $\mu_t^q$, it is also increasing in $\rho_{e,t}$ all else equal. This is a reflection of the higher expected returns (conditional on no crisis) that the experts earn when crisis risk increases.

This suggests a potentially interesting trade off: higher crisis risk means that the economy is more likely to suffer a large fall in the net worth of the financial system, but conditional on there not being a crisis we should expect expert net worth to increase faster, because the fear of crises pushes up conditional expected returns.

# 4 Unanticipated Crises

I now move on to analysing the equilibrium of the model and presenting results. Since the model is solved numerically, parameter choices are important, and I thus calibrate the model. I calibrate the model to provide a reasonable description of the data outside of crises times, as I detail below. However, given the rarity of financial crises I do not calibrate the model to match exact properties of financial crisis data. Instead, I provide results for a range of calibrations (specifically, a range of crisis frequencies) and present results across this range. In this section I consider the model with what I call "unanticipated crises". This is the solution to the model where I set $\rho_e = 0$ so that the sunspot places no weight on selecting a crisis. Nonetheless, the model will still occasionally venture into the region where crises are possible. The model will then never select the crisis equilibrium, and agents will correctly anticipate this in their expectations. Solving the model like this first has the advantage of allowing me to investigate what forces drive the model into states of the world where crises are possible without interference from the effects of crisis fear on the equilibrium itself.

## 4.1 Calibration

I choose the following parameters for the baseline calibration. One unit of time is set to one year. I set $\rho_h = 0.05$ to generate an annual risk free rate of 5%. The deprecation rates are both set to $\delta = \underline{\delta} = 0.05$. Expert productivity is set to $a = 0.1$, which can be considered a normalisation.[10] The remaining non-crisis parameters are set according to the following calibration strategy. Firstly, I attempt to match the following three moments of the US post-war data: 1) a mean quarterly growth rate of logged, HP-filtered[11] output of 0.46%, 2) a standard deviation of quarterly, logged, HP-filtered output of 0.0167, and 3) a standard deviation of the quarterly, logged, HP-filtered investment-to-capital ratio of 0.0692. The sources for these data are detailed in the data appendix of my second chapter. I choose parameters such that the model without crises ($\rho_e = 0$) generates moments close to these data. Finally, I target that the model with $\rho_e = 0$ spends 10% of its time in states where

[10] This measures the number of output goods produced from one unit of capital, and thus appropriate scalings of the other model parameters can be found such that different values of $a$ correspond to different definitions of the "number" of output goods.

[11] I use the standard smoothing parameter of 1600, corresponding to quarterly data.

crises become possible. This part of the calibration is somewhat arbitrary, but ensures that the model is able to generate crises, which is necessary for my exercise.

I parameterise the investment adjustment cost function as quadratic:

$$\iota_t = \Phi_t + \frac{\kappa}{2}\left(\Phi_t - \bar{\Phi}\right)^2 \tag{34}$$

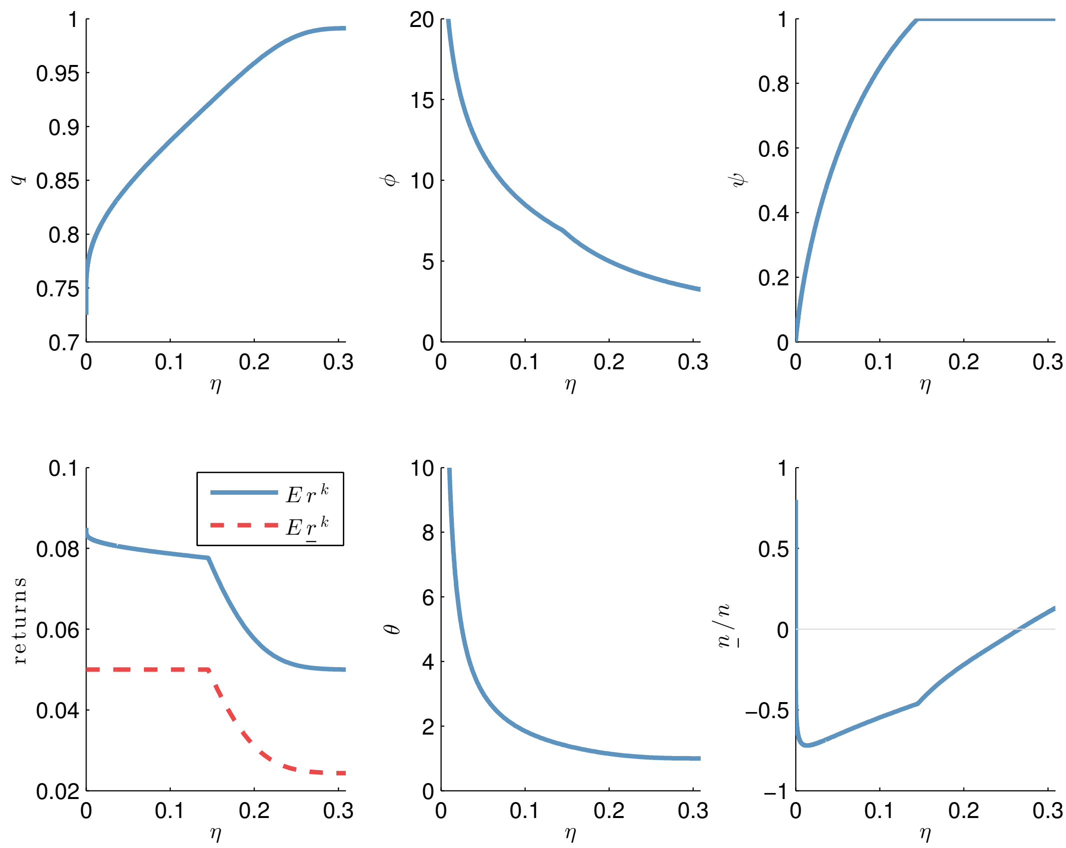
$\kappa$ measures the degree of adjustment costs ($\kappa = 0$ corresponds to no adjustment costs), and $\bar{\Phi}$ is the reference investment rate away from which adjustment costs are paid, which I take to be the average investment rate. I set $\bar{\Phi} = 0.07$ to match the targeted growth rate of output, which requires an investment rate of approximately 0.07. There are four remaining non-crisis parameters: households' productivity, $\underline{a}$, experts' discount rate, $\rho_b$, the volatility of the capital quality shock, $\sigma$, and the degree of adjustment costs, $\kappa$. These are chosen to target the four data moments above, and I use a numerical minimisation routine to minimise the squared sum of deviations from the target by varying the four parameters.

This leads to the following parameter choices: $\underline{a} = 0.07456$, $\rho_b = 0.05089$, $\sigma = 0.02591$, and $\kappa = 2.1992$. These lead to the model generating the following values of the targeted moments, when calculated as they are in the data: 1) a mean quarterly growth rate of logged, HP-filtered output of 0.38%, 2) a standard deviation of quarterly, logged, HP-filtered output of 0.0174, 3) a standard deviation of the quarterly, logged, HP-filtered investment-to-capital ratio of 0.0756, and 4) the model spends 10% of its time in the region where crises are possible.

The remaining parameters to choose correspond to aspects of financial crises. For the remainder of the paper I consider the limit case of permanent crises, setting $\rho_r = 0$. This simplification allows me to quickly compute the crisis price from (9). None of the results depend qualitatively on crises being permanent: the anticipation effects of rare but permanent crisis are much the same as the effects of more frequent but shorter crises. To demonstrate this, in the appendix I provide an example of the solution to the model with non-permanent crises. As of yet I have not set the sunspot parameter, $\rho_e$, or the exogenous destruction cost, $\chi$, as they are not needed for the solution to the model with $\rho_e = 0$. I discuss their values in section 5.

Finally, it is worth noting that this calibration leads to Brunnermeier & Sannikov's (2014a) bimodal distribution effectively vanishing. Their paper argued that the economy could endogenously spend a lot of time in very low capitalisation states following bad enough sequences of technology shocks, resulting in a regime that looks a lot like a financial crisis. However, given my calibration the probability of entering this regime is extremely small (see the stationary densities in Figure 6), and the crises driven by multiple equilibria are thus effectively the only forms of crises in the model.

Figure 1: Selected variables, model with unanticipated crises




*Solution to the model with $\rho_e = 0$. $\eta$ gives expert net worth as a fraction of the total value of the capital stock. In all panels, $\eta^*$ is the rightmost limit of the x-axis.*

### 4.2 Unanticipated Crises: Proximate Causes & Crisis-Prone Region

Figure 1 presents the numerical solution to selected variables from the model in the baseline calibration. As expert capitalisation ($\eta$) falls, experts intermediate less of the capital stock ($\psi$ falls) which causes the capital price to fall as more of it is held by unproductive households. As the price of capital falls returns increase for both experts and households. This encourages experts to increase their leverage ($\phi$ increases).

The bottom right panel of Figure 1 plots $\underline{n}_t/n_t$, and crises are thus possible whenever this line is negative (of course, given that $\rho_e = 0$ agents think that crises will never be selected in equilibrium). Crises become possible whenever expert capitalisation, $\eta$, is low enough. To understand this result, recall that $\underline{n}_t/n_t$ being negative is equivalent to

$$1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right) < 0 \tag{35}$$

Since $1 - \underline{q}/q_t < 0$, higher leverage pushes us towards crises being possible, because banks have

increased their exposure to the fall in $q_t$ but financed this using fixed debt. For low values of $\eta$, where banks are relatively undercapitalised, leverage is high in equilibrium and this gives the model a tendency to predict crises as $\eta$ falls.

On the other hand, crises are also only possible if the fall in asset prices is large enough, i.e. $\underline{q}/q_t$ is small. Graphically a crisis is a jump from the current value of $\eta$ down to $\eta = 0$, and so the price falls from its current value down to the value at the intersection with the vertical axis. For low values of $\eta$ asset prices are already low, meaning that they don't have as far to fall in a crisis. This means that for a low enough $\eta$ crises stop being possible. However, for this calibration this region is very small, to the extent that it is not visible in the plot.

The economy naturally gravitates towards higher values of $\eta$ in the absence of shocks, and spends most of its time around the higher values of $\eta$ near $\eta^*$. Hence in this calibration the economy is immune from crises near the steady state, and only becomes susceptible to crises if the experts become undercapitalised. This happens if a sequence of negative capital quality shocks erode expert net worth.

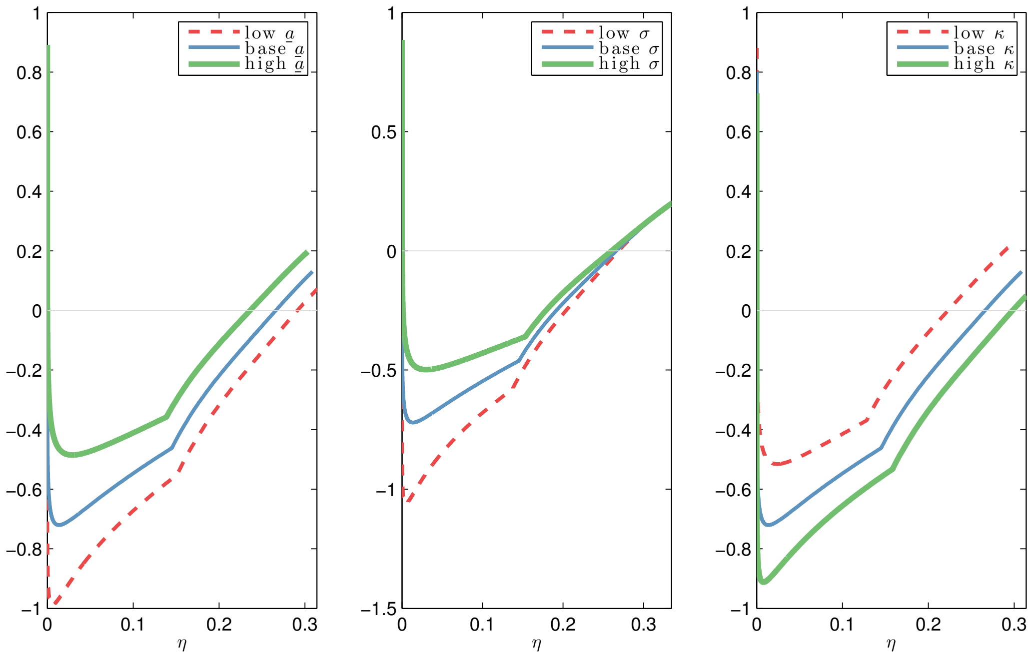
What does this tell us about the model where $\rho_e > 0$ and we actually do select crises in equilibrium? A crisis will happen in the crisis region, which the above logic tells us we will reach following a bad sequence of negative capital quality shocks. Once we are there, a crisis occurs if the "bad" sunspot is drawn, in which case the experts go bankrupt following a self-fulfilling fall in asset prices. In a crisis, $\eta$ jumps from its current value down to zero, since aggregate expert net worth falls to zero.

## 4.3 Parameter Sensitivity: Fundamental Causes of Crises

Having established that crises become possible when leverage is high and asset prices have far enough to fall, and identified the region of the state space where crises are possible, I now turn to their fundamental causes. What I mean by this is I conduct parameter sensitivity to understand what kind of economies are more or less prone to financial crises. I conduct sensitivity for three key parameters, which all have important and economically interesting effects on the size of the crisis region.

The first panel of Figure 2 plots $1 - \phi_t\left(1 - \frac{\underline{q}}{q_t}\right)$ across the state space for three different values of household productivity, $\underline{a}$. The other parameter values are all held at their baseline values. The thin, blue line in all three panels is the baseline calibration, and the dashed red and thick green lines are for a lower and higher value of the parameter respectively. For $\underline{a}$ we see that, holding all else constant, increasing $\underline{a}$ reduces the size of the crisis region, and decreasing $a$ increases it. This corresponds directly to reducing and increasing the probability of being in the crisis region, as can

Figure 2: Parameter sensitivity: Crisis net worth across changes in three parameters




*Each panel plots $\underline{n}/n$ across the state space, $\eta$. A negative value of any line means a crisis is possible at that $\eta$. Each panel calculates this variable for three values of a given parameter, holding all other parameters at their baseline values. For $\underline{a}$. the high and low values refer to 5% deviations from the baseline value, 15% for $\sigma$, and 30% for $\kappa$.*

be seen in Table 1 where I give the fraction of time spent in the crisis region under the stationary distribution for each deviation.

I showed in the previous section that crises occur because either 1) leverage is high or 2) the price of capital is high relative to the crisis price $\underline{q}$. Figure 11 in the appendix shows that it is the latter effect that is operating here. Increasing $\underline{a}$ increases the productivity of households and hence increases the fire sale price of capital. This makes crises less severe, reducing the range of states for which leverage is high enough to enable a crisis. Figure 11 shows that leverage is actually slightly higher after increasing $\underline{a}$, and hence moves in the wrong direction to explain the reduced size of the crisis region. Hence the productivity differential between $a$ and $\underline{a}$ is important in explaining the size of the crisis region because it controls how far prices have to fall in a crisis.

The second panel demonstrates the effect of fundamental uncertainty ($\sigma$) on the crisis region. Increasing fundamental uncertainty reduces the size of the crisis region, and makes entering the crisis region less likely under the stationary distribution. Figure 11 shows that, in this case, the effects of leverage and prices both work in the same direction: higher fundamental uncertainty causes experts to deleverage and reduces asset prices, both of which make the system safer from crises.

Table 1: Fraction of time spent in crisis region

| | $\underline{a}$ | $\sigma$ | $\kappa$ |
|---|---|---|---|
| *Low* | 0.26 | 0.41 | 0.03 |
| *Baseline* | 0.10 | 0.10 | 0.10 |
| *High* | 0.04 | 0.04 | 0.28 |

*Fraction of time spent in crisis region under stationary density for deviations of three parameters from baseline values when all other parameters are held at baseline. For $\underline{a}$. the high and low values refer to 5% deviations from the baseline value, 15% for $\sigma$, and 30% for $\kappa$.*

Finally, the third panel demonstrates the effect of the adjustment cost parameter $\kappa$ on the crisis region. The larger this parameter is the more costly it is to adjust your capital stock, and hence the less sensitive is investment to the capital price. Or conversely, with high values of $\kappa$ the capital price is more sensitive to investment. This means that equilibrium price function is much steeper when $\kappa$ is high, as overall investment is increased as experts get richer. Given our earlier discussion, a steep price function makes crises more likely as it increases the gap between $q$ and $\underline{q}$, even if $\underline{q}$ is fixed. The sensitivity of the crisis region to $\kappa$ was discussed in Brunnermeier & Sannikov (2014b), who interpret $\kappa$ as technological illiquidity.

The effects of $\underline{a}$ and $\sigma$ on the size of the crisis region are also interesting because they mirror earlier results in the original BrS paper. In the original model, reducing $\underline{a}$ increases the level of endogenous risk the model generates. For example, the volatility of the capital price is higher as you reduce $\underline{a}$. My result thus complements the original, by showing that not only is the volatility higher as you reduce $\underline{a}$, so is the risk of a crisis. Similarly for $\sigma$, BrS show that as you reduce the level of exogenous risk, the level of endogenous risk increases. I show that as you reduce $\sigma$ the risk of a crisis increases, complementing the original result.

# 5 Anticipated Crises

Having established the mechanics behind crises, I now move on to the general equilibrium effects of anticipating crises. At this point I need to give values for the final parameters of the model, which control various aspects of the crisis. I solve the model setting $\chi = 0$, so there are no exogenous costs of default. This choice is motivated by the fact that since in this model all experts default at the same time, any exogenous default costs would involve a large part of the economy's capital stock being exogenously destroyed. Instead, by restricting $\chi = 0$ I focus on the case where the *value* of

the economy's capital stock falls, driven by price effects.

The final parameter to set is the probability of coordinating on a crisis, $\rho_e$. This parameter controls how likely crises are in the model. Recall that in the baseline calibration crises are permanent, so $\rho_e$ can also be thought of as controlling the expected time until the economy permanently enters the crisis state. Choosing this parameter is tricky for two reasons: Firstly, financial crises are rare, so an appropriate measure of their forward-looking frequency is, by nature, a tough empirical exercise. Secondly, the model features permanent crises, and so I need to choose a relatively low probability of having a crisis to compensate for how severe the crisis state is. I settle for a value of $\rho_e = 0.1$, which implies that the expected time until the economy enters the crisis state is 357 years.[12] This is obviously extremely high, but compensates for the severity of the permanent crisis. In section 5.5 I discuss the effects of varying $\rho_e$.

### 5.1 Price & Crisis Region

Figure 3 gives the solution to key model variables for the model with anticipated crises ($\rho_e > 0$) and without ($\rho_e = 0$, which thus repeats the results of the previous section). The top left panel shows us that the price of capital is globally lower when agents anticipate that a crisis is possible. This is intuitive, since the possibility that the price will fall in the future will be reflected in a lower price today. Given that asset prices control the level of investment, this will have important consequences for growth as we shall see later. The lower middle panel plots $1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right)$. Crises are possible if this quantity is less than zero. We thus see that anticipation of crises reduces the size of the crisis region relative to the solution with $\rho_e = 0$, where agents don't anticipate crises, because the red, dashed line is negative for less of the state space than the solid blue line. This can also be seen by comparing the fraction of time spent in the crisis region, which is 0.10 for the model with $\rho_e = 0$, and falls to 0.036 for the model with $\rho_e = 0.1$.[13] Thus the model pushes back against crises once you allow agents to anticipate them. The shrinkage of the crisis region reflects either a rise in $\underline{q}/q_t$ driven by the fall in $q_t$, or a decrease in leverage. However, in the bottom left panel we see that leverage actually increases compared to the model without crises and hence the effect is driven by the fall in asset prices.

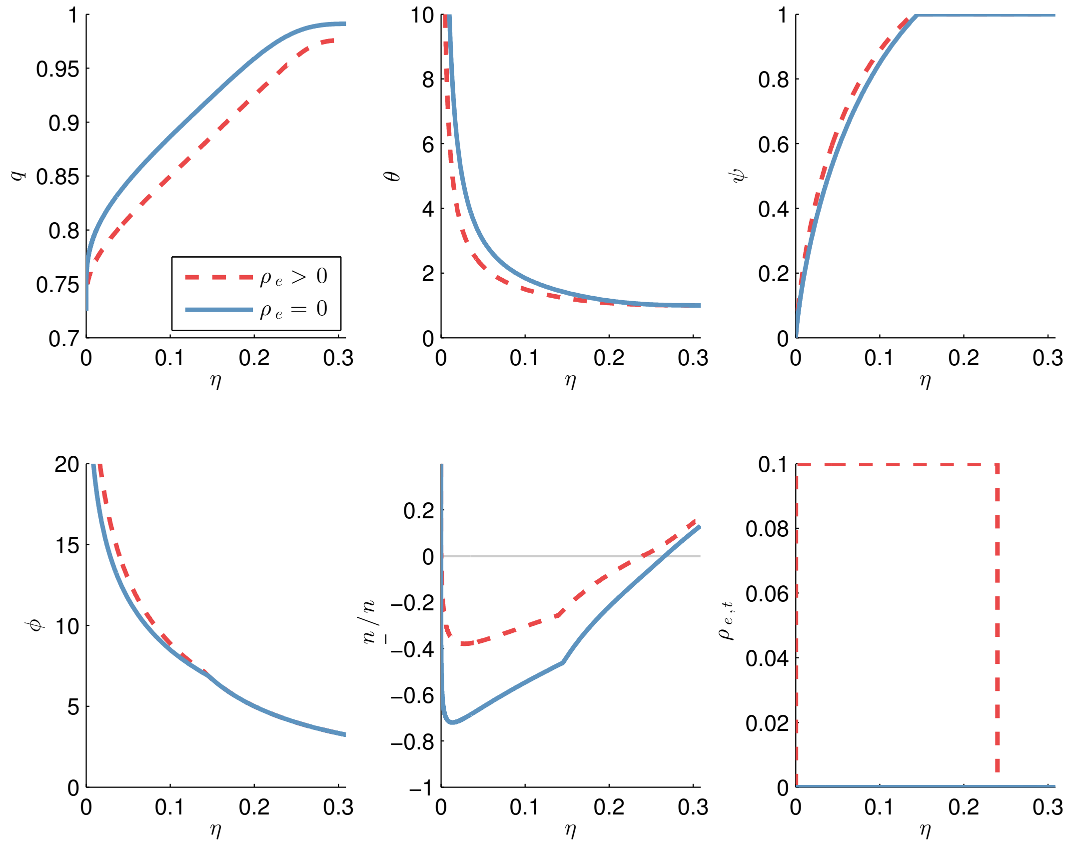
### 5.2 Leverage

Before discussing the main results, it is instructive to understand the effect that the fear of crises has on leverage. The model solution reveals that leverage is higher across the state space in the

[12] This is calculated starting from an initial value of $\eta_0 = \eta^*$. Details of the computation and simulation procedure are provided in the appendix.

[13] The fraction for the model with $\rho_e = 0.1$ is calculated using the stationary density excluding crisis realisations.

Figure 3: Model solution with and without anticipated crises. Key variables




*The dashed red line gives the solution to the model with the baseline positive value of $\rho_e$, meaning the economy will eventually experience a crisis. The solid blue line gives the solution where $\rho_e = 0$ and agents never coordinate on a crisis.*

model where agents anticipate crises. This solution is for the case without exogenous default costs ($\chi = 0$) so the as the earlier discussion of the equations in Section 3.11 suggests, we should not expect financial crises to give experts any immediate reason to deleverage. In this section I discuss the general equilibrium forces which lead to higher equilibrium leverage.

The key is that the fear of crises tends to increase expected returns *conditional on there not being a crisis* by reducing asset prices. Even though experts are exposed to the downside risk of crises, they do not take this into consideration since they know they will go bankrupt in these states anyway. Thus from their point of view the main change between the two models is the higher expected returns in the model with anticipated crises, and they leverage up to take advantage of this.

Alternatively, we can see this by looking again at equation (26), which is the equation which

determines leverage:

$$\frac{a-\underline{a}}{q_t}+\underline{\delta}-\delta-\rho_{e,t}\chi\frac{\underline{q}}{q_t}+\sigma_t^\theta\left(\sigma+\sigma_t^q\right)=0$$

Lower asset prices increase the return differential between experts and households by increasing $\frac{(a-\underline{a})}{q_t}$. This increases the advantage that experts have over households, and requires us to create a disadvantage for experts to return them both to indifference. This is done by increasing leverage, which increases the amount of endogenous risk.

Thus we see that experts lean in to crises, rather than reducing leverage to try and avoid their exposure to them. This is due, of course, to the assumptions that make avoiding crises unprofitable for experts. If we add exogenous default costs ($\chi > 0$) then it is possible that experts might deleverage in some regions of the state space, because exogenous default costs (which are only paid if capital is in the hands of experts) provide us an incentive to put capital in the hands of households instead. I present the solution for $\chi = 0.25$ in the appendix for comparison with the baseline model.

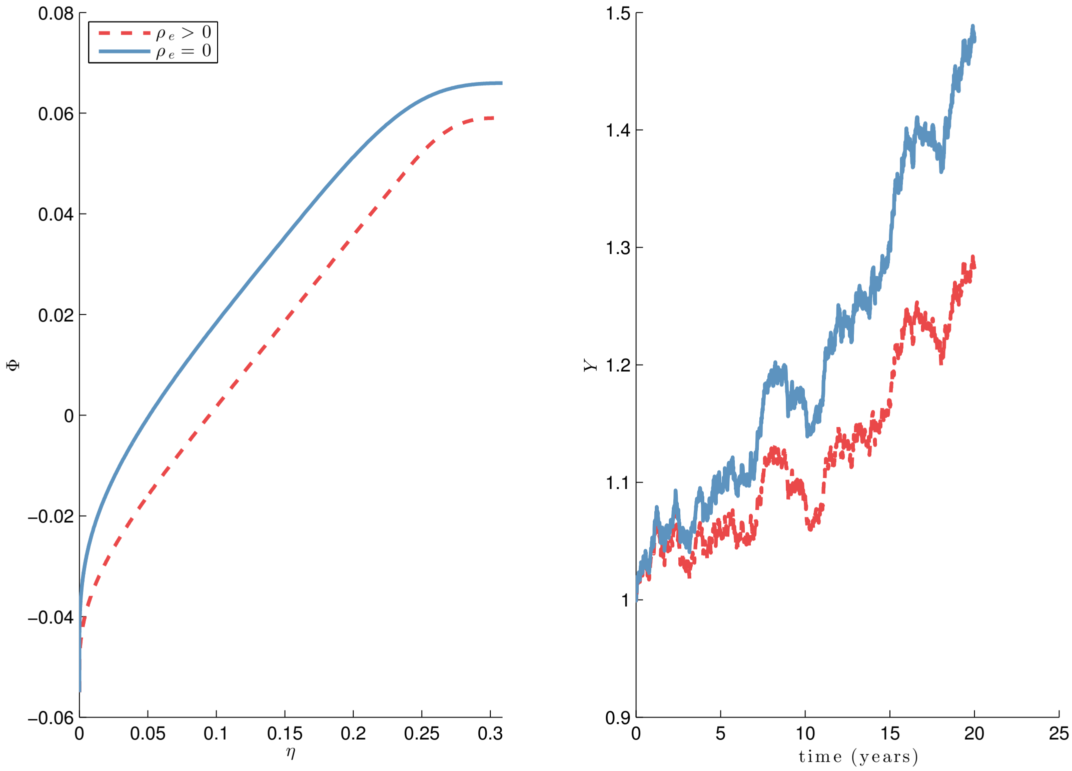
### 5.3 Investment & Growth

My first main result is the effect that financial crises have on growth. The left panel of Figure 4 plots the policy function for the investment rate, $\Phi_t$, in the models with and without anticipated crises. This picture mirrors the effect that crisis fear had on asset prices, which is unsurprising given the tight link between asset prices and investment in the model. Note that due to the linearity of the production functions in capital, the investment rate becomes independent of the capital stock, and the model features long-run, endogenous growth. The investment rate, along with the depreciation rate, then pins down the growth rate of the capital stock according to the expert and household accumulation equations, (1) and (2). Aggregating these two equations when $\delta = \underline{\delta}$, as is true in my calibration, yields the evolution of the total capital stock:

$$dK_t = \left(\Phi(\iota_t) - \delta\right) K_t dt + \sigma K_t dZ_t \tag{36}$$

In the model where agents anticipate crises, the investment rate is everywhere lower because asset prices are always lower. Given the lower asset price, there is less incentive to invest since the (static) profits from investing are lower. Alternatively, we could could think about this as a rough discounted sum. A crisis scenario involves handing the capital stock to the household to intermediate. Given the household's low productivity this means that the discounted sum of profits from investing are lower, leading to lower investment. The growth effects of expected crises are illustrated in the second panel of Figure 4. Here I simulate the models, plotting a sample path for output ($Y_t = (a\psi_t + \underline{a}(1-\psi_t))K_t$). The widening gap between output, driven by the under-accumulation of capital in the crisis economy is clear. For the current calibration, the average quarterly growth rate of output falls from 0.38% to

Figure 4: Investment policy function & output sample path




*Left panel plots the investment policy function in the models with ($\rho_e > 0$) and without ($\rho_e = 0$) crises. Right panel plots a simulated time path for output for both economies, normalised to one in the first year for both. The path for the economy with crises is constructed so that no crises occur during the sample path.*

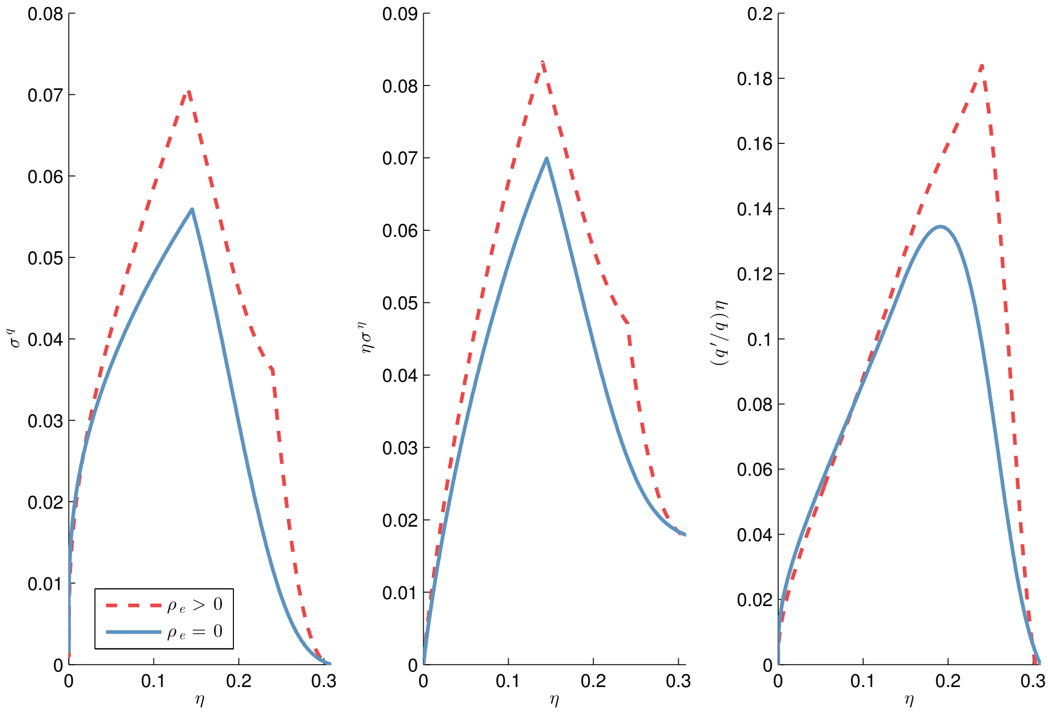
0.19%. Growth rate effects are likely to have much larger effects on welfare than level effects, and the halving of growth rates adds up over a long enough horizon. Given that other studies have focused on level effects of crises, this suggests an important alternative motivation for policy to address financial crises. Additionally, it could present an explanation for the slow recovery in the aftermath of the recent financial crisis, a time where fear of a repeat was surely escalated, as I discuss further in section 5.5.

### 5.4 The financial crisis accelerator

In this section I discuss the financial crisis accelerator. This is the result that the fear of crisis increases the endogenous volatility of the economy. Specifically, in some regions the fear of crises makes the economy more responsive to the exogenous capital quality shock. This can be seen in the first two panels of Figure 5 where the volatility terms for both $q_t$ and $\eta_t$ are plotted. These

volatilities give the impact responses to the capital quality shock $dZ_t$.

Figure 5: The financial crisis accelerator




*The first two panels plot the loading on the Brownian motion of the capital price and state variable, $\eta$. The final panel plots the dependence of $\sigma^q$ on $\eta\sigma^\eta$, as described by (38).*

The results are very state dependent: for both $q_t$ and $\eta_t$, the increase in volatility is larger in the middle of the state space. What drives this result? As explained by BrS, the financial accelerator in continuous time can be understood as the interaction between the volatility terms of $\eta_t$ and $q_t$. For example, a negative shock reduces bank net worth, reducing $\eta_t$. But reducing $\eta_t$ reduces the price $q_t$ as more capital is intermediated by inefficient households. The reduction in $q_t$ further reduces $\eta_t$, and the cycle continues. This can be seen by the interdependency between the equations for $\sigma_t^q$ and $\sigma_t^\eta$ (which are derived in the appendix):

$$\sigma_t^\eta \eta_t = \eta_t(\phi_t - 1)(\sigma + \sigma_t^q) \tag{37}$$

$$\sigma_t^q = \frac{q'(\eta_t)\sigma_t^\eta \eta_t}{q_t} \tag{38}$$

Each depends on the other, and solving the two together gives the solutions for the volatilities in (27). The slope of the price function, $q'(\eta_t)$ is important because this tells us how much prices are going to fall in response to a marginal fall in $\eta_t$. A steep price function thus gives a severe financial accelerator because prices fall a lot in response to a fall in net worth, making the secondary effect

on net worth larger. The other determinant of the size of the multiplier is the current leverage of the experts, $\phi_t$. High leverage means that experts have a large exposure to $q_t$, and makes their net worth more sensitive to changes in asset prices, worsening the financial accelerator.

The changes in the volatility terms between the two solutions can thus be understood by appealing to the changes in leverage and the slope of the price function caused by the fear of crises. As previously mentioned, the model with anticipated crises has higher leverage than the model without, which thus contributes to the increase in volatility. The changes in the slope of the price function are plotted in the last panel of Figure 5. In the central region where the volatilities are increased most relative to the model without crises, we see that the slope of the price function is increased.

This region overlaps closely with the region where crises are possible. As we move deeper in to the crisis region we expect to remain there for longer, placing more and more downwards pressure on asset prices because they might suddenly fall in a crisis. This makes $q_t$ very sensitive to our position in the state ($\eta_t$) around this region. This is the essence of the financial crisis multiplier: shocks that push the economy closer to (or deeper into) the crisis region will push down asset prices a lot as agents anticipate a possible crash, making the standard financial accelerator more powerful.

It is only in and near to the crisis region that the financial crisis accelerator emerges. This also creates an intuitive asymmetry in the model: starting from the steady state, the financial accelerator is worse in response to negative shocks than it is to positive shocks. This is because a series of negative shocks bring us closer to the crisis region, prompting asset prices to fall faster, harshly eroding net worth and so on. In response to positive shocks we move further away from the crisis region. The probability of crisis in the near future was already close to zero, and remains so, leading to smaller changes in asset prices and a smaller financial accelerator.

Table 2 gives the standard deviations of output and the investment rate across several values of $\rho_e$. This reveals that the financial crisis accelerator effect is rather large for the investment rate (and hence also for asset prices) and quantitatively less important for output itself. In particular, going from the baseline value of $\rho_e$ down to zero more than halves the volatility of investment, but only reduces the volatility of output by around 3%.

Finally, it is worth noting that these results again echo results in the original BrS paper. In their paper they show that the financial accelerator can be made quantitatively more powerful by understanding that their model is prone to occasional prolonged periods of financial distress. The model features a bimodal stationary distribution, where a sufficiently bad series of exogenous shocks can lead to the economy getting trapped with low net worth (low $\eta_t$) for a long time. This possibility is what allows asset prices to fall a lot in response to negative shocks, as agents anticipate that this outcome becomes more likely. My result thus complements theirs, because I show that their powerful

financial accelerator can also be rationalised by appealing to crises of a self-fulfilling nature.

### 5.5 Stability & growth

In this section I demonstrate how varying the probability of coordinating on a crisis affects equilibrium. Clearly, reducing this probability will, by construction, reduce the likelihood of crises in the model. It also has intuitive effects on the growth rate and volatility of the economy. Table 2 gives various moments of the model across a range of values for $\rho_e$.

Table 2: Effects of varying the probability of coordinating on a crisis

| | $\rho_e = 0$ | $\rho_e = 0.05$ | $\rho_e = 0.01$ | $\rho_e = 0.02$ | $\rho_e = 0.05$ | $\rho_e = 0.1$ |
|---|---|---|---|---|---|---|
| $TTC$ | $\infty$ | 1301 | 904 | 629 | 428 | 357 |
| $g_y$ | 0.38% | 0.35% | 0.32% | 0.27% | 0.21% | 0.19% |
| $\sigma_y$ | 0.0174 | 0.0174 | 0.0175 | 0.0176 | 0.0178 | 0.0180 |
| $\sigma_{I/K}$ | 0.0756 | 0.0804 | 0.1071 | 0.1079 | 0.1492 | 0.1711 |

*$TTC$ is the expected time time until the economy experiences a crisis. $g_y$ refers to the average growth rate of quarterly GDP, $\sigma_Y$ its standard deviation, and $\sigma_{I/K}$ the standard deviation of the quarterly investment ratio, $I_t/K_t$.*

The first column gives the solution to the model where agents never coordinate on crises ($\rho_e = 0$), and successive columns increase $\rho_e$, up to the baseline value in the final column. The first row gives the expected time until the economy enters the crisis state, which is infinity by construction when $\rho_e = 0$, and falling as the probability of coordinating on a crisis is increased. The second row gives the effect on the average quarterly growth rate of output, which is decreasing as the probability of coordinating on a crisis is increased. The final two rows give the effects on the volatilities of output and the investment rate, which are increasing as the probability of coordinating on a crisis is increased, demonstrating the financial crisis accelerator.

These results also suggest an interesting behavioural interpretation of the model. If agents' fears of a future crisis exogenously increase after experiencing a crisis (modelled by increasing $\rho_e$) then the economy will grow slowly, but be less volatile and less susceptible to crises in the immediate aftermath. If these fears then decrease if the economy doesn't experience a crisis, growth will gradually recover, and the economy will start becoming more volatile and more susceptible to crises again as time goes on.

# 6 Policy

I now turn to the policy implications of my model. I focus first on prudential (ex-ante) policies which aim to reduce crisis risk by limiting expert leverage. I then discuss ex-post bailout which aim to reduce the ex-ante perception of crisis risk by signalling the government's commitment to maintaining asset prices.

## 6.1 Prudential policy: leverage constraints

### 6.1.1 Minimally active leverage constraint

Since crises in my model are only possible for high enough expert leverage, policies which limit leverage ex ante are natural candidates for ruling out, or reducing the probability of, financial crises. Remember that a crisis is only possible at time $t$ if equilibrium leverage ($\phi_t$) and asset prices ($q_t$) satisfy

$$1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right) < 0 \tag{39}$$

Thus if the government imposes the following regulatory leverage constraint it can completely rule out the possibility of financial crises:

$$\phi_t \leq \bar{\phi}_t^{ma} \equiv \frac{1}{\left(1 - \frac{\underline{q}}{q_t}\right)} \tag{40}$$

I call this constraint the "minimally active" leverage constraint. It is active in the sense that it requires active monitoring and adjustment by the regulator: the leverage constraint depends on today's capital price and the capital price during a crisis. It is minimal in the sense that this is the loosest leverage constraint which completely rules out crises. Quantitatively, this constraint says that if asset prices are known to drop by a fraction $1/x$ during a crisis, leverage cannot be higher than $x$. So if asset prices are thought to drop by a quarter, leverage would be restricted to be no higher than 4.

How does the minimally active leverage constraint affect equilibrium? Trivially, it rules out financial crises. The question that remains is how does it affect the other features of equilibrium? This will be crucial in determining whether or not the policy is welfare improving. I will focus on two key aspects of equilibrium: volatility and growth. As discussed in the introduction, the prevailing view of the effects of prudential leverage constraints is that they would reduce volatility, but at the expense of reducing growth. I show that this is not true in my model.

The details of the model solution with a leverage constraint are relegated to the appendix, and I present only the results here. Figure 6 plots selected figures from the model solution with the minimally active leverage constraint, as well as the original model solutions with and without crisis

risk. Given the typical discourse surrounding prudential policies, the results are surprising: the average growth rate of quarterly output rises to 0.38% after the introduction of the policy. This rise, from the original 0.19% of the model with crisis, brings the growth rate of the economy all of the way back up to the growth rate of the economy with no crisis risk. Hence ruling out crisis comes with the benefit of higher growth, not the cost of lower growth.

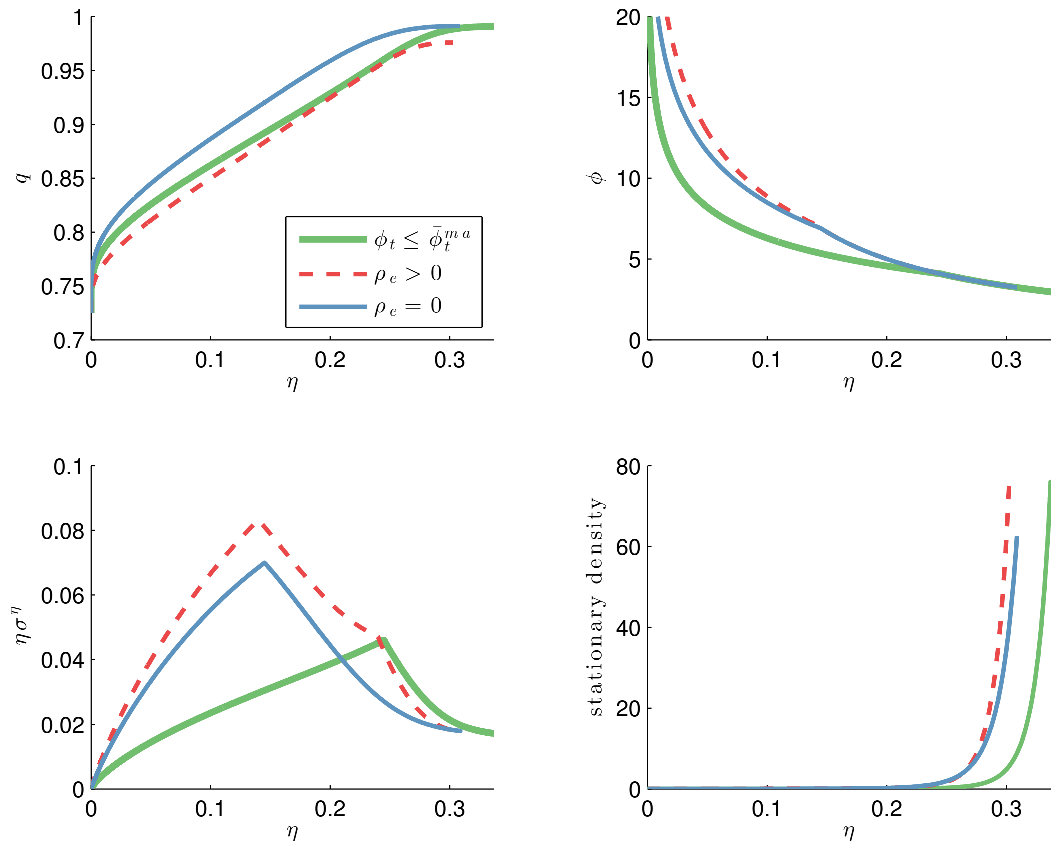
We can understand why by inspecting Figure 6. The top left panel plots the price of capital across the three models. For high levels of expert capitalisation, $\eta$, the minimally active leverage constraint increases the price of capital almost all the way from the its original price (dashed red line) to the price in the model without crisis risk (thin blue line). For lower values of $\eta$ the benefits of the policy on asset prices are smaller, however, the economy spends very little time in this region under the stationary distribution. The intuition for the increase in prices is simple: by ruling out crises the policy removes the possibility of of prices jumping down in the future to $\underline{q}$, which increases prices today. Combined with the shifting right of the stationary distribution past the distribution of the model without crises this leads to the increase in average prices.

The top right panel plots leverage across the three models, showing that the leverage constraint, by construction, only has large effects on equilibrium leverage in the central region where crises were originally possible. General equilibrium effects lead to small changes outside of this region. Of course, this raises the question of how experts are able to fund increased investment while having their leverage reduced. Consider a region of the state space where experts hold all of the capital stock. Then the definition of expert leverage implies that $q_t K_t = \phi_t N_t$. For a given capital stock, this shows us that higher asset prices can only be supported following a reduction in leverage if expert net worth increases *more* than the fall in leverage. The investment first order condition implies that investment is fully tied down by the price of capital, and hence that investment and growth can also only increase if equity rises sufficiently.

Thus an increase in equity, compensating for lower debt, is key to generating increased growth following the implementation of a regulatory borrowing constraint. Is this a reasonable thing to expect? In the model, experts cannot raise equity, and the increase in equity is thus funded by experts paying out net worth (as consumption) less often. This implies that the policy actually *raises* the value to experts of retaining earnings. This is an intuitive idea: if experts know a crisis is coming they have incentive to consume now in order to consume their net worth before it is lost in a crisis. Hence leverage policy could encourage equity issuance by making the financial sector safer, reducing the incentive to withdraw equity as dividends instead of investing it. This can be seen in the bottom right panel, which shows that expert capitalisation is higher under the policy than without it, since the stationary density is shifted to the right.

Finally, the bottom left panel plots the volatility of the aggregate state, $\eta$ across the three models. In the central region, the leverage constraint reduces volatility relative to both the models with and without crises. From the discussion in Section 5.4, this decrease is a direct consequence of the decrease in leverage, which reduces the financial accelerator in that region. However, this reduction is not across the whole state space, and outside of this region there is actually a small increase in volatility. This is due to the increased slope of the price function in certain regions, which leads to a slight exacerbation of the financial accelerator.

Figure 6: Effects of the minimally active leverage constraint




*Model solution in models with (dashed red) and without (thin blue) crises, as well as model with minimally active leverage constraint (thick green). Stationary density in the model with crises is calculated ignoring crisis realisatons.*

Putting all of these effects together makes a strong case for the minimally active leverage constraint in this model: it rules out financial crises, increases growth, and reduces volatility in most regions of the state space. With this in mind, I now turn to looking at the effects on welfare of the policy. The total welfare of experts at a given time is $W_t^e \equiv \eta_t \theta_t q_t K_t$ and that of households

is $W_t^h \equiv (1-\eta_t)q_tK_t$.[14] Since the model features two classes of agents there is no single welfare criterion that we can use, so I first examine the impact of the policy on the welfare of each group individually. As a total welfare criterion I select total welfare $W_t \equiv W_t^e + W_t^h$.

**Proposition 2.** *Any policy which increases the price of capital, $q_t$, on impact increases household welfare if households are holding capital ($\psi_t < 1$) and leaves household welfare constant otherwise. Holding the other constant, policies which increase $q_t$ or $\theta_t$ on impact increase expert welfare, and expert welfare increases iff $\theta_t' (1 - \phi_t (1 - q_t'/q_t)) > \theta_t$ where primed variables are post policy. Following the unanticipated implementation of a policy, $q_t$ and $\eta_t$ immediately jump to the values which solve:*

$$q_t' = q_{pol}(\eta_t') \tag{41}$$

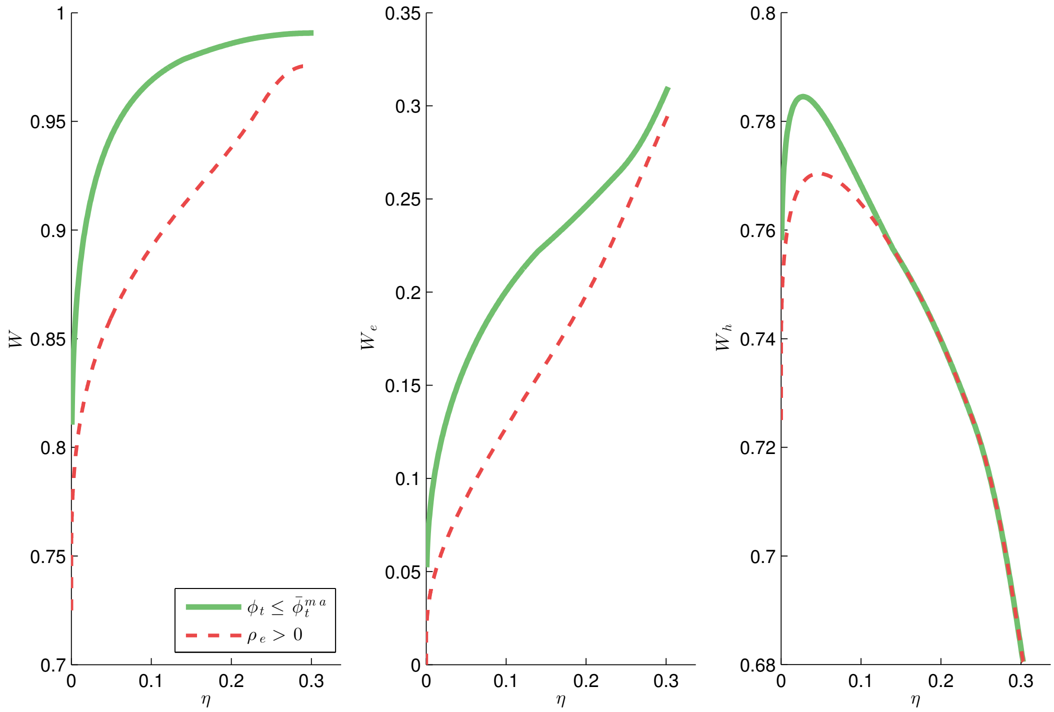
$$\eta_t' = \left(1 - \phi_t \left(1 - \frac{q_t'}{q_t}\right)\right) \frac{q_t}{q_t'} \eta_t \tag{42}$$

The proofs are simple algebra and are omitted. The last part of the proof points out that the implementation of any policy will have an immediate impact on prices, and hence on expert capitalisation. Thus when we evaluate the welfare impact of a policy, we need to take into account two things: Firstly, equilibrium welfare as a function of the state $\eta_t$ in the new policy regime. Secondly, today's state $\eta_t$ and how that translates into our state $\eta_t'$ after the policy is implemented. Welfare varies across the state space, and so whether or not implementing the leverage policy improves welfare looking forward could in principle depend on the state of the economy today. To address this, in Figure 7 I plot $W_t$, $W_t^e$ and $W_t^h$ across the state space in the model with crises and the model with the minimally active leverage constraint, normalising $K_t$ to one. Thus it is important to note that these are welfare changes *on impact* given the current state of the world, and take into account agents expectations of the future evolution of the economy under the equilibrium distribution of the state.

The figure reveals that, regardless of the state today, the policy increases both total welfare and weakly increases the welfare of both sets of agents. It is hence Pareto improving across the whole state space, as well as being a policy that both sets of agents would support. This is because crises hurt both agents, and they are both happy to see them removed. Crises mean that experts will eventually go bankrupt, losing their ability to generate net worth. They also mean that households, who will eventually have to intermediate capital, will suffer as the economy operates at lower productivity. Notice that towards the right of the state space, where households don't hold capital, they are indifferent about the implementation of the policy on impact. This is because in this region they do

[14] To see this for experts note that an individual expert's maximised value is $\theta_t n_t$, and that the total net worth of the expert sector is $\eta_t q_t K_t$. Since households are risk neutral their welfare is just their net worth, which totals $(1-\eta_t)q_tK_t$.

Figure 7: Welfare: minimally active leverage constraint




*The left panel plots $W(\eta)$ across the state space in the model with crisis risk, and the associated welfare on impact if we are in state $\eta$ today and the policy is implemented. Hence the thick green line plots $W_{pol}(\eta'(\eta))$ where $\eta'$ is calculated for every $\eta$ as in Proposition 2. If the solid green line is above the dashed red line the policy improves welfare on impact. The centre and right panels plot the same for $W_e$ and $W_h$ respectively.*

not hold any capital, and hence do not realise any increase in their net worth on impact. Since their welfare is simply their net worth, they also do not realise any increase in their welfare.[15]

This result, that the minimally active leverage constraint increases welfare for both agents, is a direct consequence of thinking about the costs arising from financial crises. To see this, note that the same policy actually reduces welfare if it is implemented in an economy without financial crises ($\rho_e = 0$). Of course, this suggests an intuitive condition for the minimally active leverage constraint to increase welfare: crises must be *sufficiently likely*. In particular, there is a threshold likelihood of financial crises above which the policy increases welfare, and below which it does not.

[15] Of course, their welfare could change over time as the economy evolves, and will indeed evolve differently with and without policy. However, this is taken into account when computing their current welfare, since it is a forward looking measure.

### 6.1.2 Implementation issues

One issue with the minimally active leverage constraint is that while it improves welfare, it requires a lot of information for the government to implement it correctly. It requires knowledge of current asset prices *and* how far asset prices would fall in a crisis. To address these issues, in this section I investigate how deviations from this policy affect its effects. In particular, I consider a government who attempts to implement the minimally active leverage constraint, but instead accidentally implements:

$$\phi_t \leq \bar{\phi}_t^x \equiv \frac{x}{\left(1 - \frac{\underline{q}}{q_t}\right)} \tag{43}$$

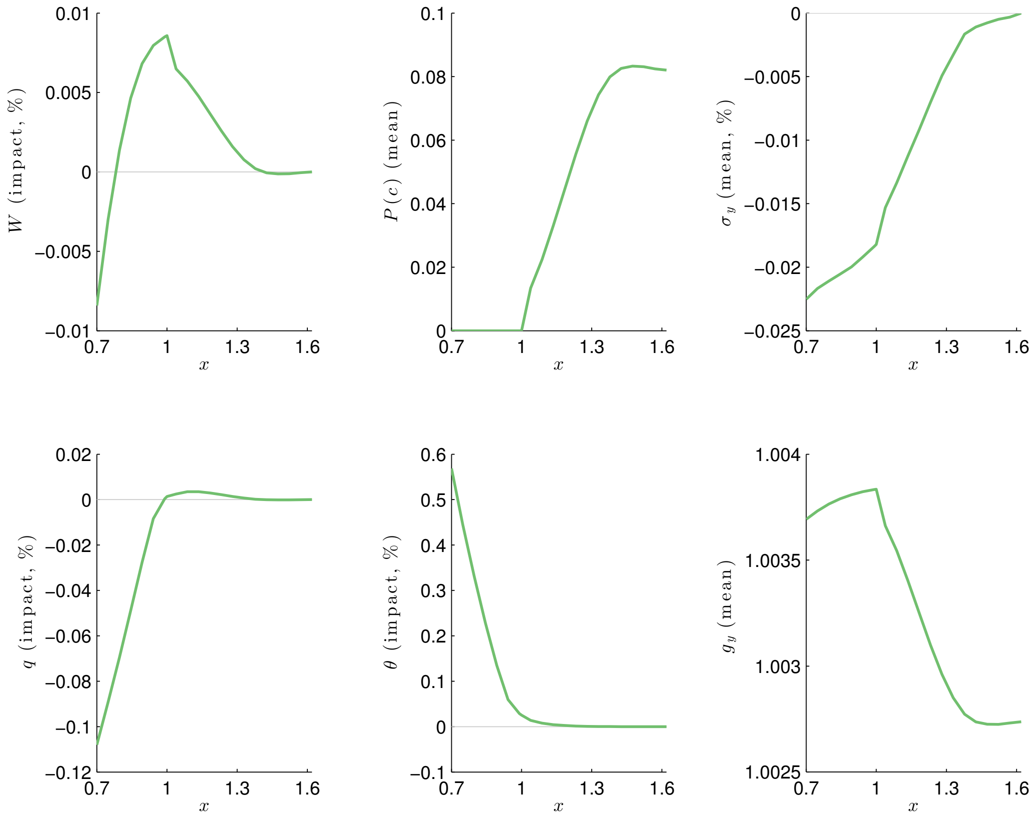
For some $x > 0$. This policy is essentially a slightly tighter ($x < 1$) or looser ($x > 1$) version of the minimally active leverage constraint. The policies which are tighter deliver no extra benefit in reducing the probability of there being a crisis, since this has already been driven to zero. I plot the effect on policy of selected variables across a range of values of $x$ in Figure 8. Since I am now comparing across many model solutions, I restrict myself to focusing on the effect of policy for a specific starting value of the state today, which I take to be the ergodic mean under the model with crisis and without policy.

The top left panel shows that the improvement in total welfare is maximised (within this class of policies) by choosing the minimally active leverage constraint: $x = 1$. Note that at the value of $\eta$ at which I am making these comparisons, households do not hold any capital, and hence see no welfare gains. Thus the entire welfare gain is driven by an increase in the welfare of experts. Policies which are slightly too tight ($x$ less than but close to 1) still improve welfare, but by less. This reflects the additional distortions that are introduced by leverage constraints which are too tight. However, for these parameter values the policymaker has room for error before policy becomes actually harmful, and policies can be up to 20% too tight and still deliver welfare benefits. In the other direction, policies which are looser than the minimally active constraint still deliver welfare benefits, but they are again smaller.[16] If the policy is so loose that it does not bind at all then the policy trivially delivers no welfare benefits.

The bottom left and centre panels decompose the welfare gain into its $q_t$ and $\theta_t$ components. This reveals that the welfare cost of overly tight policies derives from an instantaneous lower asset prices (which are a reflection of the lower present value of output). This is partially offset by increases in the value of net worth to experts, which reflects the gain in instantaneous profits they can make as arbitrage is restricted.

[16] Note that there is a discontinuous jump up in welfare as $x$ approaches one from above. This is due to the nature of the exercise. Any value of $x$ which is an $\varepsilon$ above one has a positive probability of experiencing a crisis, which converges to a number other than zero as $\varepsilon \to 0$. $x = 1$ then delivers a probability of crises of exactly zero.

Figure 8: Effects of badly implemented policy, selected variables




*All variables computed from the same value of $\eta$, which I take as the ergodic mean of the model without policy. Variables denoted "impact, %" are impact changes, computed in fractional deviations from the value without policy. Variables denoted "mean" are main values from simulations of the model with the corresponding policy (ignoring crisis realisations). $P(c)$ denotes the fraction of time spent in the crisis region.*

The remaining three panels show how the various policies affect the moments of the economy. The top centre panel shows that tighter policies reduce the fraction of time the economy spends in the crisis region, which eventually falls to zero when $x \leq 1$. The top right panel shows that tighter policies reduce the volatility of output. The bottom right panel shows that, starting from a high $x > 1$, tighter constraints increase average output growth. But average growth is maximised for $x = 1$, and further tightening of the constraint starts to erode the gains to growth.

Overall, these results highlight an interesting *inverse-$U$* shape in the welfare gains from prudential leverage constraints. Policies which are too loose trivially deliver little or no welfare gains. Intermediate policies deliver welfare gains by ruling out crises, and this benefit is maximised once the probability of having a crisis is reduced to zero. Beyond this point, extra tightness is welfare

reducing since it distorts the intermediation of capital without delivering any extra crisis-reduction benefits. The model thus emphasises a role for leverage policy, but also stresses caution in its use.

### 6.2 Ex-post policy: bailouts

Another commonly discussed and contentious policy instrument is ex-post bailouts. The idea behind bailouts is that the fundamental problem during crises is a lack of net worth in the financial system, and bailouts aim to fix this by directly injecting net worth (either for free or at a discounted rate). Bailouts are criticised mainly on ex-ante incentive grounds, with the argument being that they incentive risk taking by reducing the punishment banks face when everything goes wrong. In my baseline model this trade off does not exist, and bailouts can be effective at completely ruling out crises without imposing any incentive distortions. The result is summarised in the following proposition:

**Proposition 3.** *Suppose that in the event of a crisis each expert is recapitalised to their original level of net worth, $n_t$. Then crises are not possible, and the model equilibrium is identical to the solution without crises (i.e. with $\rho_{e,t} = 0$).*

*Proof.* The recapitalisation policy rules out crises because it rules out jumps in net worth, and hence $\eta_t$: if it were to jump the recapitalisation policy simply jumps us right back to where we started. Since jumps don't happen in equilibrium, experts never receive any bailouts, and the model equations are identical to those with $\rho_{e,t} = 0$. □

This result is very similar in spirit to the original Diamond & Dybvig (1983) result that (in their baseline model) deposit insurance can improve allocations. In my model, bailouts promise to restore asset prices and net worth to their original level in the event of a crisis, completely removing the possibility of a crisis even happening because now agents have no reason to coordinate on the bad equilibrium. Since crises now never happen in equilibrium, experts can never receive any bailouts, which means that their incentives cannot be distorted by the possibility of receiving bailouts.

Of course, as in Diamond & Dybvig (1983), the result that this policy does not induce any distortions is special and due to the simplicity of the setup. Specifically, since bailouts are never needed in equilibrium (just as deposit insurance is never used in their model) there is no incentive to change behaviour to try and receive this bailout. Thus we would expect less successful or well targeted bailout policies, which actually led to bailouts being given in equilibrium (as they are in the real world) to induce ex-ante distortions. For this reason we should remain sceptical of bailouts, and future work explicitly assessing the pros and cons is necessary. Indeed, as I have previously discussed, my technological assumption could be viewed as a stand in for other frictions, such as bailouts, which encourage banks to allow themselves to get in to trouble during a crisis.

### 6.3 Market based solutions

One possibility which the literature has started to address is that instead of government policies placing limits on the behaviour of the financial sector, the government could encourage the formation of markets to deal with the specific externalities involved.

The typical financial accelerator paper, including this one, assumes that lending is not contingent on aggregate state variables. This is what gives the accelerator power, since following aggregate shocks the value of assets can change dramatically, while the value of debt is fixed. If debt was allowed to be state contingent then the value of debt can also adjust to offset the change in the value of assets, protecting net worth and blunting the accelerator. Dmitriev & Hoddenbagh (2013) show that under the optimal (state contingent) contract, the financial accelerator disappears in the standard Bernanke, Gertler & Gilchrist (1999) model. In Clymo (2015) I show this is also true in the context of a model with a Gertler & Kiyotaki (2010) style borrowing constraint. Carlstrom, Fuerst, Ortiz & Paustian (2014) take an agnostic view on the degree of indexation of debt, and perform a structural estimation to pin down the value in the context of their model, again finding that higher indexation reduces the financial accelerator. Finally, Kilenthong & Townsend (2014) argue for market based solutions to price externalities in a general theoretical framework.

The takeaway from this literature is that if we are to believe in the power of financial frictions, we need to be confident that markets are sufficiently less than complete. While this may ultimately be an empirical question, I provide some additional theoretical insights here in the context of my model. The model solved above features only defaultable debt, which gives a less than fully state contingent set of assets to trade on. I show in this section that if we add a second "insurance" asset, which pays off during a crisis, then the same frictions which make anticipated crises possible also mean that no individual expert would be willing to take out insurance against the possibility of a crisis. This result this stands in contrast to the results above, and highlights a limit on the power of market based solutions.

In particular, consider the following insurance asset. If the asset is held from $t$ to $t+dt$ and there is no financial crisis, the holder pays a premium $r_t^I dt$. If there is a crisis then the asset pays out one unit of the consumption good. This is a classic insurance contract over the event of a financial crisis happening. The household provides this insurance contract to the experts, who may choose any non-negative amount of insurance.

**Proposition 4.** *In the baseline model, an expert will never choose to hold positive amounts of the insurance asset, and the equilibrium with insurance is identical to the equilibrium with only defaultable debt.*

The proof is relegated to the appendix, and I discuss the intuition here. This insurance contract

allows an expert to transfer wealth between future states of the world: do I want money tomorrow if things work out, or if there is a crisis? However, if anticipated crises are to exist in equilibrium, we require frictions which make the value of wealth to an expert during a crisis, $\underline{\theta}$, low. In fact, in the baseline model $\underline{\theta} = 1$, its lowest possible value, because the aggregate capital externality reduces an expert's ability to produce if all other experts are bankrupt. This ensures that experts are willing to take on high enough leverage to allow themselves to go bankrupt during a crisis. What Proposition 4 establishes is that under the frictions which allow crises to happen in the baseline model, the addition of an additional insurance market is unable to provide any extra protection. The intuition is simple: a crisis is not a good time to have net worth, so experts have no incentive to use insurance to transfer wealth to that state of the world.

Of course, the above result relies as crucially on the assumption of extra frictions as does the very existence of crises in my model. The point is that the conditions that make anticipated crises possible in my model are the very conditions which make the above market based solution infeasible.

# 7 Conclusion

In conclusion, I study the ex ante effects of the fear of future financial crises. I show theoretically that this "crisis fear" has both negative growth and business cycle effects. Financial crises push capital away from experts and towards less productive households, worsening the allocation of capital. Thus the possibility of future crises lowers the expected return on capital. This lowers asset prices, investment and growth today, even if experts are currently well enough capitalised to survive a crisis. The model features endogenous growth, leading to permanent growth effects of crises on growth. The externality that generates endogenous growth is also crucial for generating crises, by reducing the productivity of surviving experts in crises and hence encouraging them to overleverage and allow crises to happen in equilibrium. The possibility of future crises also creates a state-dependent "financial crisis accelerator" in which shocks which push the economy closer to crisis lead to more severe financial accelerator effects than those that push the economy away from crisis.

The model has implications for policy, and shows that explicitly taking into account agents' expectations that there could be future crises can overturn the received wisdom about the tradeoffs of prudential policy. In particular, in my model restrictions on expert leverage can remove the possibility of financial crises and simultaneously increase growth. This is in contrast to the standard view that leverage constraints should reduce growth by restricting the ability of the financial sector to intermediate funds. While this effect still operates in my model, leverage constraints also encourage growth by making the system safer and promoting investment. This strengthens the case for prudential policy, and future quantitative work should address the importance of this effect relative

to the traditional growth-harming effects of prudential policy.

## References


[1] Acharya, Viral V, Tanju Yorulmazer, 2007. *Too many to fail – An analysis of time-inconsistency in bank closure policies.* Journal of Financial Intermediation, Volume 16, p1-31.

[2] Admati, Anat & Hellwig, Martin, 2013. *The Bankers' New Clothes: What's Wrong With Banking and What to Do About It.* Princeton University Press, February 24, 2013.

[3] Bank for International Settlements, 2010. *An assessment of the long-term economic impact of stronger capital and liquidity requirements.* Basel Committee on Banking Supervision, August 2010.

[4] Bernanke, Ben, 2012. *The Federal Reserve's Response to the Financial Crisis, Lecture 3.* George Washington University School of Business, March 27, 2012. Available at http://www.federalreserve.gov/newsevents/files/chairman-bernanke-lecture3-20120327.pdf. Last accessed: 21/4/2015.

[5] Bernanke, Ben S, Gertler, Mark & Gilchrist, Simon, 1999. *The financial accelerator in a quantitative business cycle framework.* Handbook of Macroeconomics, in: J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, pages 1341-1393 Elsevier.

[6] Brunnermeier, Markus K, and Yuliy Sannikov, 2014a. *A Macroeconomic Model With A Financial Sector.* American Economic Review 104.2: 379-421.

[7] Brunnermeier, Markus K, and Yuliy Sannikov, 2014b. *International Credit Flows And Pecuniary Externalities.* Working paper.

[8] Carlstrom, Charles T & Fuerst, Timothy S, 1997. *Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis.* American Economic Review, American Economic Association, vol. 87(5), pages 893-910, December.

[9] Carlstrom, Charles T, Fuerst, Timothy S, Ortiz, Alberto, Paustian, Matthias, 2014. *Estimating Contract Indexation in a Financial Accelerator Model.* Journal of Economic Dynamics and Control, Volume 46, September 2014, Pages 130-149.

[10] Cheng, Ing-Haw, Sahil Raina, and Wei Xiong, 2014. *Wall Street and the Housing Bubble.* American Economic Review, 104(9): 2797-2829.

[11] Claessens, Stijn, & M. Ayhan Kose, 2013. *Financial Crises: Explanations, Types, and Implications.* IMF Working Paper WP/13/28. January 2013.

[12] Clymo, Alex, 2015. *Labour Market Matching, Stock Prices & the Financial Accelerator.* Mimeo.

[13] Diamond, D., and Dybvig, P., 1983. *Bank Runs, Deposit Insurance, and Liquidity.* Journal of Political Economy 91, 401-419.

[14] Dmitriev, Mikhail & Hoddenbagh, Jonathan, 2013. *The Financial Accelerator & the Optimal Lending Contract.* Mimeo.

[15] Duchin, Ran, Denis Sosyura, 2014. *Safer Ratios, Riskier Portfolios: Banks' Response to Government Aid.* Journal of Financial Economics (JFE), 113, 1-28.

[16] Eggertsson, Gauti B. & Paul Krugman, 2012. *Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach.* The Quarterly Journal of Economics, Oxford University Press, vol. 127(3), pages 1469-1513.

[17] Ennis, Huberto and Keister, Todd, 2003. *Economic Growth, Liquidity and Bank Runs.* Federal Reserve Bank of Richmond Working Paper No. 03-01.

[18] Farhi, Emmanuel, and Jean Tirole. 2012. *Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts.* American Economic Review, 102(1): 60-93.

[19] Financial Times, 2007. *Citigroup chief stays bullish on buy-outs.* Available at http://www.ft.com/cms/s/0/80e2987a-2e50-11dc-821c-0000779fd2ac.html#axzz3NknxFGvy. Published: July 9, 2007, 10:08 pm. Last accessed: 4/1/14.

[20] Foote, Christopher L, Kristopher S. Gerardi and Paul S. Willen, 2012. *Why Did So Many People Make So Many Ex Post Bad Decisions? The Causes of the Foreclosure Crisis.* Federal Reserve Bank of Atlanta Working Paper 2012-7.

[21] Gertler, Mark & Nobuhiro Kiyotaki, 2010. *Financial Intermediation and Credit Policy in Business Cycle Analysis.* Handbook of Monetary Economics, Volume 3, 2010, Pages 547-599.

[22] Gertler, Mark & Nobuhiro Kiyotaki, 2013. *Banking, Liquidity and Bank Runs in an Infinite-Horizon Economy.* NBER Working Papers 19129, National Bureau of Economic Research, Inc.

[23] Guardian, 2014. *The next crisis could be here soon.* Economics blog, Larry Elliott. http://www.theguardian.com/business/economics-blog/2014/jan/20/next-financial-crisis-here-soon-history. Published: 20/01/2014. Last accessed: 11/10/14

[24] Guerrieri, Veronica and Lorenzoni, Guido, 2011. *Credit Crises, Precautionary Savings and the Liquidity Trap.* Working paper.

[25] Imbs, Jean, 2007. *Growth and volatility.* Journal of Monetary Economics, Volume 54, Issue 7, October 2007, Pages 1848-1862.

[26] Jermann, Urban, and Vincenzo Quadrini. 2012. *Macroeconomic Effects of Financial Shocks.* American Economic Review, 102(1): 238-71.

[27] Khan, Aubhik, Julia K. Thomas, 2013. *Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity.* Journal of Political Economy, University of Chicago Press, vol. 121(6), pages 1055 - 1107.

[28] Kilenthong, Weerachart T, & Townsend, Robert M, 2014. *A Market Based Solution to Price Externalities: A Generalized Framework.* NBER Working Paper No. 20275.

[29] Kocherlakota, Narayana R. 2009. *Bursting Bubbles: Consequences and Cures.* Manuscript, Federal Reserve Bank of Minneapolis.

[30] Koudijs, Peter and Hans-Joachim Voth, 2014. *Leverage and Beliefs: Personal Experience and Risk Taking in Margin Lending.* NBER Working Paper No. 19957, March 2014.

[31] Mailath, George J, Loretta J. Mester, 1994. *A Positive Analysis of Bank Closure.* Journal of Financial Intermediation, Volume 3, Issue 3, June 1994, Pages 272-299.

[32] Martin, Alberto and Jaume Ventura, 2014. *Managing Credit Bubbles.* NBER Working Paper No. 19960, March 2014.

[33] McDonald, Robert & Paulson, Anna (2015). *AIG In Hindsight* NBER Working Paper No. 21108, April 2015.

[34] Osilia, Una Okonkwo, Anna Paulson, 2014. *Crises and confidence: Systemic banking crises and depositor behavior.* Journal of Financial Economics, Volume 111, Issue 3, March 2014, Pages 646-660.

[35] Perri, Fabrizio, & Quadrini, Vincenzo , 2014. *International Recessions.* Working paper.

[36] Ramey, Garey and Valerie A. Ramey, 1995. *Cross-Country Evidence on the Link Between Volatility and Growth.* The American Economic Review, Vol. 85, No. 5 (Dec., 1995), pp. 1138-1151.

[37] Reinhart, Carmen, & Rogoff, Kenneth, 2009. *This Time is Different. Eight Centuries of Financial Folly.* Princeton University Press.

[38] Reinhart, Carmen, & Rogoff, Kenneth, 2012. *Sorry, U.S. Recoveries Really Aren't Different.* Bloomberg. http://www.bloomberg.com/news/articles/2012-10-15/sorry-u-s-recoveries-really-aren-t-different. Published: 10/15/2012. Last accessed: 14/05/2015.

[39] Romer, Paul M, 1986. *Increasing Returns and Long-run Growth.* Journal of Political Economy, University of Chicago Press, vol. 94(5), pages 1002-37, October.

[40] Schroth, Enrique, Gustavo A. Suarez, Lucian A. Taylor, 2014. *Dynamic debt runs and financial fragility: Evidence from the 2007 ABCP crisis.* Journal of Financial Economics, Volume 112, Issue 2, May 2014, Pages 164-189.

[41] Wall Street Journal, 2012. *Sowing Seeds of the Next Major Crisis.* Current Account, Francesco Guerrera. http://online.wsj.com/news/articles/SB10001424052702304818404577347641050572260. Published: 16/04/2012. Last accessed: 11/10/14.

# Appendices

## A Endogenous growth

We can consider the linear production functions as the reduced form of a simple endogenous growth model. In particular, consider the experts' production function $y_t = ak_t$. Suppose instead that experts produce according to $y_t = z\hat{K}_t^{1-\alpha}k_t^{\alpha}l_t^{1-\alpha}$, where $l_t$ is their labour choice, and $\hat{K}_t$ is aggregate expert capital, which an individual takes as given. There is thus a capital externality: experts don't take into account that their capital choice affects the productivity of other experts. Experts hire labour at wage $w_t$. Assume that households inelastically supply one unit of labour to experts, and one unit to households.

An expert chooses labour to maximise static profit: $\pi_t = z\hat{K}_t^{1-\alpha}k_t^{\alpha}l_t^{1-\alpha} - w_t l_t$. This yields the first order condition $w_t = (1-\alpha)z\hat{K}_t^{1-\alpha}k_t^{\alpha}l_t^{-\alpha}$. After optimising labour, an expert's profit function becomes linear in individual capital:

$$\pi_t = \left((1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}\right) w_t^{\frac{\alpha-1}{\alpha}} z^{\frac{1}{\alpha}} \hat{K}_t^{\frac{1-\alpha}{\alpha}} k_t \tag{44}$$

Imposing market clearing ($l_t = 1$) and $\hat{K}_t = k_t$ this profit becomes $\pi_t = \alpha z k_t$. Given the linearity of both the individual and equilibrium profit function in $k_t$, we see that this model is isomorphic to the baseline BrS model with $a = \alpha z$, and where output ($y_t$) is replaced with profit ($\pi_t$). Doing the same with the household production function yields the same result, with $\underline{a} = \alpha\underline{z}$. Thus we are able to reinterpret BrS' model as an endogenous growth model based on Romer (1986) under certain parameter restrictions.[17]

## B Derivations

### B.1 Household derivations

#### B.1.1 Risky debt

I derive the required interest rate in discrete time and take the limit. One unit is lent today, and next period $1 + r_t dt$ is repaid unless there is default. If there is default the expert's assets are seized and

[17] The main restriction is setting the exponent on $\hat{K}_t$ equal to the labour share. As discussed in Ennis & Keister (2003) this restriction yields linear production, which means that aggregate capital does not have to be considered a state variable and removes transitional dynamics from the capital stock. Additionally, my assumption that labour is supplied inelastically to each class of agents is a simplification. The assumption removes interactions between the two groups through the wage, and can be removed at the expense of making the reduced form productivities effectively dependent on the aggregate state.

split amongst the lenders. The expert will have assets worth $(1-\chi)q_{t+dt}k_{t+dt}$ where $\chi$ is destroyed. The expert borrowed $d_{t+dt} = q_t k_{t+dt} - n_t$, so the assets which can be seized per unit lent is

$$(1-\chi)\frac{\underline{q}_{t+dt}k_{t+dt}}{q_t k_{t+dt} - n_t} = (1-\chi)\frac{\frac{\underline{q}_{t+dt}}{q_t}\phi_t}{\phi_t - 1}$$

Where $\phi_t \equiv q_t k_{t+dt}/n_t$. Suppose we are in a region where crises are possible. Then the expert defaults if the bad sunspot is drawn. A good sunspot is drawn with probability $P_g = e^{-\rho_{e,t}dt} \simeq 1 - \rho_{e,t}dt$, and a bad with probability $P_b = 1 - e^{-\rho_{e,t}dt} \simeq \rho_{e,t}dt$. The household discounts the future between $t$ and $t+dt$ with factor $\beta = e^{-\rho_h dt} \simeq 1 - \rho_h dt$. The expected return on risky debt must equal:

$$1 = \beta P_g(1 + r_t dt) + \beta P_b \mathrm{E}_t \left[ (1-\chi)\frac{\frac{\underline{q}_{t+dt}}{q_t}\phi_t}{\phi_t - 1} \middle| f_{t+dt} = 1 \right] \tag{45}$$

Where the expectation term is the expectation conditional on there being a crisis at $t+dt$. This expectation is for the different values of $q_{t+dt}$ we might have depending on the value of the other shocks to the economy, and I denote the price by $\underline{q}_{t+dt}$ to make it clear that this is the crisis price. Taking the limit as $dt \to 0$, using the approximations above and noting that $dt^2 = 0$:

$$1 = 1 - \rho_h dt - \rho_{e,t}dt + r_t dt + \rho_{e,t}dt(1-\chi)\frac{\underline{q}_t}{q_t}\frac{\phi_t}{\phi_t - 1} \tag{46}$$

Where I have also used that $\underline{q}_{t+dt} = \underline{q}_t + d\underline{q}_t$ and $d\underline{q}_t$ is of order $dt$. Rearranging and dividing by $dt$:

$$r_t = \rho_h + \rho_{e,t}\left(1 - (1-\chi)\frac{\underline{q}_t}{q_t}\frac{\phi_t}{\phi_t - 1}\right) \tag{47}$$

Note in the special case of full destruction, $\chi = 1$, we have simply $r_t = \rho_h + \rho_{e,t}$. Also runs are only possible if:

$$\bar{N} = N_t - q_t k_{t+dt} + \underline{q}_t k_{t+dt} < 0 \Rightarrow \phi_t > \frac{1}{1 - \frac{\underline{q}_t}{q_t}} \tag{48}$$

and in this region $r_t > \rho_h$, i.e. the expert pays a premium for default risk. As leverage increases the interest rate increases, reaching a maximum of $\rho_h + \rho_{e,t}\left(1 - (1-\chi)\frac{\underline{q}_t}{q_t}\right)$.

## B.2 Expert derivations

Conjecture that marginal value follows:

$$d\theta_t = \mu_t^\theta \theta_t dt + \sigma_t^\theta \theta_t dZ_t + df_t\left(\underline{\theta}_t - \theta_t\right) \tag{49}$$

Where $\underline{\theta}_t$ is an expert's marginal value following a crash, which is to be determined. Experts' value can be expressed as:

$$\rho_b \theta_t n_t = \max_{dC_t \geq 0, \phi_t \geq 0} \{dC_t + \mathrm{E}_t d(\theta_t n_t)\} \tag{50}$$

Using Ito's lemma the last term becomes:

$$\mathrm{E}_t d(\theta_t n_t) = \mathrm{E}_t\left[d\theta_t n_t + \theta_t dn_t + dn_t d\theta_t + df_t\left(\underline{\theta}_t \underline{n}_t - \theta_t n_t\right)\right] \tag{51}$$

Giving:

$$(\rho_b + \rho_{e,t})\theta_t n_t = \max_{dC_t \geq 0, \phi_t \geq 0} \{dC_t + \mathrm{E}_t [d\theta_t n_t + \theta_t dn_t + dn_t d\theta_t] + \rho_{e,t} \underline{\theta}_t \underline{n}_t\} \tag{52}$$

Where it is understood that the jump terms are excluded from $d\theta_t$ and $dn_t$ in the above equation. $dn_t$ follows:

$$dn_t = \left(dr_t^k \phi_t + (1 - \phi_t) r_t - dc_t\right) n_t \tag{53}$$

Remember that there is implicitly a jump here contained in $dr_t^k$. Also define $dc_t = dC_t/n_t$. Excluding the jump term from $dr_t^k$ and using (4) and (11) we can write $dn_t$ as

$$\frac{dn_t}{n_t} = \left(\left(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right) dt + (\sigma + \sigma_t^q) dZ_t\right) \phi_t + ...$$

$$... + (\rho_h + \rho_{e,t})(1 - \phi_t) dt + \rho_{e,t}(1 - \chi)\frac{\underline{q}_t}{q_t}\phi_t dt - dc_t \tag{54}$$

Note here that the interest rate $r_t$ is calculated assuming that the expert has taken on enough leverage to go bankrupt during a crisis. If the expert takes on low enough leverage she can survive a crisis, in which case she only pays interest $r_t = \rho_h$ and the equation above is the same just setting $\rho_{e,t} = 0$. Let's calculate expectations of the moments conditional on $df_t = 0$:

$$\mathrm{E}_t dn_t = \left[\left(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)\phi_t + (\rho_h + \rho_{e,t})(1 - \phi_t) + \rho_{e,t}(1 - \chi)\frac{\underline{q}_t}{q_t}\phi_t - \frac{dc_t}{dt}\right] n_t dt \tag{55}$$

$$\mathrm{E}_t d\theta_t = \mu_t^\theta \theta_t dt \tag{56}$$

$$\mathrm{E}_t dn_t^2 = (\sigma + \sigma_t^q)^2 \phi_t^2 n_t^2 dt \tag{57}$$

$$\mathrm{E}_t dn d\theta = \sigma_t^\theta (\sigma + \sigma_t^q) \phi_t n_t \theta_t dt \tag{58}$$

$$\mathrm{E}_t d\theta_t^2 = \left(\sigma_t^\theta\right)^2 \theta_t^2 dt \tag{59}$$

Plugging these in, and dividing by $n_t$:

$$(\rho_b + \rho_{e,t})\theta_t = \max_{dc_t \geq 0, \phi_t \geq 0} dc_t + ...$$

$$\mu_t^\theta \theta_t dt + ...$$

$$\left[\left(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)\phi_t + (\rho_h + \rho_{e,t})(1 - \phi_t) + \rho_{e,t}(1 - \chi)\frac{\underline{q}_t}{q_t}\phi_t - \frac{dc_t}{dt}\right]\theta_t dt + ...$$

$$\sigma_t^\theta (\sigma + \sigma_t^q) \phi_t \theta_t dt + ...$$

$$\rho_{e,t}\underline{\theta}_t \frac{\underline{n}_t}{n_t} \tag{60}$$

Let's assume for now that $\frac{\underline{n}_t}{n_t} = 0$, i.e. that the expert allows herself to go bankrupt during a crisis. In general, it actually depends on $\phi_t$. I discuss this in the next section, where we consider the conditions under which experts will allow themselves to go bankrupt during a crisis. The leverage first order condition gives:

$$\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q - (\rho_h + \rho_{e,t}) + \rho_{e,t}(1-\chi)\frac{\underline{q}_t}{q_t} = -\sigma_t^\theta (\sigma + \sigma_t^q) \tag{61}$$

And the first order condition for consumption gives:

$$\theta_t \geq 1 \tag{62}$$

with equality if $dc_t > 0$. At the optimum, and when $dc_t = 0$, evaluating the value function gives:

$$\rho_b \theta_t = \mu_t^\theta \theta_t dt + \rho_h \theta_t dt + \rho_{e,t}\underline{\theta}_t \frac{\underline{n}_t}{n_t} \tag{63}$$

And since $\underline{n}_t = 0$ this becomes:

$$\mu_t^\theta = \rho_b - \rho_h \tag{64}$$

### B.3 Will an expert optimally allow herself the risk of going bankrupt?

In this section I discuss under what conditions an expert will allow herself to take on enough leverage such that she would go bankrupt during a crisis. Indeed, this is necessary for anticipated crises to be possible in equilibrium. To do this we need to delve a bit deeper into the crisis value term $\underline{\theta}_t \frac{\underline{n}_t}{n_t}$. There is a kink here since the expert has limited liability. If the expert goes bankrupt this value must drop to zero because the expert's total net worth is wiped out, but if the expert chooses low enough leverage she will survive, and this value will be positive:

$$\underline{\theta}_t \frac{\underline{n}_t}{n_t} = \begin{cases} \underline{\theta}_t \left(1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right)\right) & : 1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right) \geq 0 \\ \underline{\theta}_t \cdot 0 = 0 & : 1 - \phi_t \left(1 - \frac{\underline{q}}{q_t}\right) < 0 \end{cases} \tag{65}$$

If she takes on low enough leverage she also only pays the risk free rate on her borrowing, so her net worth evolves according to:

$$\frac{dn_t}{n_t} = \left(\left(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right) dt + (\sigma + \sigma_t^q)\, dZ_t\right) \phi_t + ...$$

$$... + \rho_h (1 - \phi_t)\, dt - dc_t \tag{66}$$

There is a clear cost-benefit decision here: you can take on high leverage, in which case you make large profit as long as there is no crisis, but large losses during a crisis, or you can take on low

leverage and make low profit in normal times and less losses in a crisis. A key variable is how much a unit of net worth is worth to an expert during a crisis: $\underline{\theta}_t$. If a unit of net worth is worth a lot during a crisis (as we might expect, since returns are high) then this will push experts towards caution. If the expert is happy with a level of leverage in the region where she doesn't go bankrupt during a crisis then we can show that the following FOC holds:

$$\frac{a-\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q - \rho_h - \rho_{e,t}\left(1 - \frac{\underline{q}_t}{q_t}\right)\underline{\theta}_t + \sigma_t^\theta\left(\sigma + \sigma_t^q\right) = 0 \tag{67}$$

Compare this to the leverage FOC in the region where she does go bankrupt:

$$\frac{a-\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q - \rho_h - \rho_{e,t}\left(1 - \frac{\underline{q}_t}{q_t}\right) - \rho_{e,t}\chi\frac{\underline{q}_t}{q_t} + \sigma_t^\theta\left(\sigma + \sigma_t^q\right) = 0 \tag{68}$$

This shows us the differences between the costs of increasing leverage on the margin in the two regions. If the expert has low enough leverage to survive a crisis (first equation), increasing leverage hurts because it increases the losses in a crisis, which are valued at the marginal value of net worth during a crisis. If the expert has already chosen high enough leverage to go bankrupt, then increasing leverage by an extra unit hurts in a different way: it increases borrowing costs.

**Proposition 5.** *If* $(\underline{\theta}_t - 1)\left(1 - \frac{\underline{q}_t}{q_t}\right) \leq \chi\frac{\underline{q}}{q_t}$ *then experts find it optimal to choose high enough leverage to go bankrupt during a crisis.*

*Proof.* Suppose that a crisis is possible in equilibrium. Then in equilibrium we know that (68) holds because this is the required optimality condition for the other experts to be willing to accept a crisis. We need to verify that an individual expert is willing to choose leverage that leads her to go bankrupt during a crisis. To verify this it is sufficient to check that lowering leverage does not increase today's value. Small changes in leverage lead to no change in value, because remember that experts are locally indifferent about their leverage choices in equilibrium. But what about a change large enough to avoid bankruptcy? This is not profitable as long as (67) holds either with equality, or instead has the terms on the left hand side greater than zero. This means that in this region increasing leverage weakly increases value, and given the linearity this means the expert increases leverage all the way in to the region where she does go bankrupt. Mathematically, we require:

$$\frac{a-\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q - \rho_h - \rho_{e,t}\left(1 - \frac{\underline{q}_t}{q_t}\right)\underline{\theta}_t + \sigma_t^\theta\left(\sigma + \sigma_t^q\right) \geq 0 \tag{69}$$

Substituting in (68) and cancelling terms leaves:

$$(\underline{\theta}_t - 1)\left(1 - \frac{\underline{q}_t}{q_t}\right) \leq \chi\frac{\underline{q}}{q_t} \tag{70}$$

$\square$

The intuition behind the result is quite simple. Recall that $\underline{\theta}_t$ is the marginal value of net worth during a crisis. (70) says that this value cannot be too high, otherwise experts would want to deleverage (all the way to zero leverage, in fact) to take advantage of this. This benefit of reducing leverage must be weighed against the cost, which is the lost expected revenue from lending. In equilibrium this can be derived from the optimality of the other experts, giving the above expression. In the baseline model, with $\chi = 0$, this condition requires that $\underline{\theta}_t = 1$. This is a very strong requirement, which follows from the way the model is constructed. In particular, recall that since experts don't face leverage constraints in equilibrium they must be indifferent about their leverage choices. Given their risk neutrality, this means that they derive no utility from any of their lending, which is why $\underline{\theta}_t$ has to be so low in order to convince them to lend even in the face of a potential crisis. This would not be true in a richer model, in which experts derived more explicit benefits from lending, where $\underline{\theta}_t$ would be allowed to be higher.

The capital externality which creates endogenous growth actually also ensures that $\underline{\theta}_t = 1$ in the baseline model, making crises possible in equilibrium. This is summarised in the following proposition:

**Proposition 6.** *In the basline model with endogenous growth (as described in Appendix A) and with permanent financial crises ($\rho_r = 0$), $\underline{\theta}_t = 1$.*

*Proof.* From an individual expert's point of view all of the other experts go bankrupt in a crisis, and therefore are unable to hold any capital. Since an individual expert has zero mass this means that the total capital held by experts, $\hat{K}_t$ is zero. Due to the production externality, this means that the productivity of a surviving expert is zero in a crisis: $y_t = z\hat{K}_t^{1-\alpha} k_t^{\alpha} l_t^{1-\alpha} = 0$. Since the expert is unable to produce, now or for the rest of time, she might as well consume her net worth, leading to $\underline{\theta}_t = 1$. $\square$

The idea behind this admittedly stylised assumption is that the disruption in financial markets during a crisis would make it hard for a surviving bank to function efficiently. It also matches the empirical fact that the value of being a surviving bank (as measured by the market value of bank equity) appears to be very low: In the US bank equity values fell by an average of 80% during the crisis and have remained persistently low. It is also worth noting that this result does not hold exactly for non-permanent crises, because then experts' net worth can have higher value, even if they cannot produce today. This is because they might want to hold on to their wealth in order to benefit from positive returns when the other experts are bailed out, and $\hat{K}_t$ becomes positive again. It is easy to show that, for any $\hat{\eta}$, $\underline{\theta}_t$ is falling in the expected length of the crisis, so there is always a long enough crisis (small enough $\rho_r$) to ensure that (70) holds.

To see the importance of the aggregate-capital externality in allowing crises, it is instructive to think about the case where the reduced form production functions ($y_t = ak_t$ and $\underline{y}_t = \underline{a}\underline{k}_t$) are the true production functions, and there is thus no feedback from aggregate expert capital to individual expert productivity:

**Proposition 7.** *Consider a model without the aggregate-capital externality, meaning that $y_t = ak_t$ and $\underline{y}_t = \underline{a}\underline{k}_t$ are the true individual expert and household production functions respectively. Then $\underline{\theta}_t = \infty$.*

*Proof.* To see this, note that while the economy is in the crisis capital is priced by the household, at $\underline{q}$, which is constant (until the exogenous recapitalisation restores the economy to positive $\eta$). Since $\underline{q}$ and $\underline{\theta}$ are constant there is also effectively no risk to the surviving expert from investing (the expert only cares about covariance risk of net worth with $\theta_t$). There also cannot be another financial crisis while we are in a crisis, by construction, so the expert can borrow risk free at $r_t = \rho_h$. Since the household is investing, we know that $\mathrm{E}_t d\underline{r}_t^k = \rho_h$, and using the definitions of $d\underline{r}_t^k$ and $dr_t^k$ we can show that

$$\mathrm{E}_t dr_t^k = \rho_h + \left( \frac{a - \underline{a}}{\underline{q}} + \underline{\delta} - \delta \right) dt > r_t = \rho_h$$

I.e. the expected profit from increasing leverage is positive ($\mathrm{E}_t dr_t^k - r_t > 0$). Without any risk there is no force that creates a cost of leverage to experts, and a suviving expert would thus choose infinite leverage, making infinite instantaneous profit, leading to $\underline{\theta}_t = \infty$. □

This proposition highlights the fundamental issue making it hard to generate expected financial crises in this model: without any other frictions, it is great to be the only surviving expert in a financial crisis. With asset prices so low you can make huge amounts of profit. The aggregate-capital externality powering endogenous growth is a way to shut this down, by making it bad to be the only surviving expert. Other more realistic assumptions could replace this, but the general idea is that disruptions in financial markets during a crisis should reduce the value of being a surviving expert. For example, imposing a borrowing constraint during crises would reduce the ability of experts to take advantage of the temporarily high returns. Another possibility is that the expectation of bailouts is what leads experts to allow themselves to get in trouble during a crisis. Indeed, this is the focus of several theoretical papers, for example Farhi & Tirole (2012), Acharya & Yorulmazer (2007), and Mailath & Mester (1994). Indeed, this concern is empirically validated, as shown by Duchin & Sosyura (2014) who use the TARP program to show that individual banks increase the riskiness of their portfolios in response to signals that they might receive government aid in the future. Future work could incorporate this mechanism as a potential rationalisation of crises in my model.

### B.4 Equilibrium derivations

#### B.4.1 Derivation of $\mu_t^q$, $\sigma_t^q$, $\mu_t^\theta$, and $\sigma_t^\theta$

Using Ito's lemma on $q_t = q(\eta_t)$:

$$dq_t = q'(\eta_t)d\eta_t + \frac{1}{2}q''(\eta_t)d\eta_t^2 + \left(\underline{q} - q_t\right) df_t \tag{71}$$

Using the conjectured law of motion for $\eta_t$, (19), gives:

$$dq_t = \left(q'(\eta_t)\mu_t^\eta\eta_t + \frac{1}{2}q''(\eta_t)\left(\sigma_t^\eta\right)^2\eta_t^2\right)dt + q'(\eta_t)\sigma_t^\eta\eta_t dZ_t + \left(\underline{q} - q_t\right) df_t \tag{72}$$

Equation coefficients from the above equation with the conjectured law of motion for $q_t$, (3), gives:

$$\mu_t^q = \frac{q'(\eta_t)\mu_t^\eta\eta_t + \frac{1}{2}q''(\eta_t)\left(\sigma_t^\eta\right)^2\eta_t^2}{q_t} \qquad \sigma_t^q = \frac{q'(\eta_t)\sigma_t^\eta\eta_t}{q_t}$$

We can do the same exercise for $\theta_t$ to calculate $\mu_t^\theta$ and $\sigma_t^\theta$ as:

$$\mu_t^\theta = \frac{\theta'(\eta_t)\mu_t^\eta\eta_t + \frac{1}{2}\theta''(\eta_t)\left(\sigma_t^\eta\right)^2\eta_t^2}{\theta_t} \qquad \sigma_t^\theta = \frac{\theta'(\eta_t)\sigma_t^\eta\eta_t}{\theta_t}$$

#### B.4.2 Derivation of $\mu_t^\eta$ and $\sigma_t^\eta$

Now I need to use Ito's lemma multiple times to work out the evolution of $\eta_t$ using the definition $\eta_t \equiv N_t/(q_tK_t)$. I first need the individual evolutions of $N_t$, $q_t$ and $K_t$. $q_t$ is already given by (3). $K_t$ is total capital ($K_t \equiv K_t^b + K_t^h$) which it is easy to show evolves via:

$$\frac{dK_t}{K_t} = \left(\Phi(\iota_t) - \delta\psi_t - \underline{\delta}(1-\psi_t)\right)dt + \sigma dZ_t \tag{73}$$

This comes from aggregating (1) and (2). The evolution of $N_t$, total bank capital, is just the aggregate version of (53):

$$\frac{dN_t}{N_t} = dr_t^k\phi_t + \left(\rho_h + \rho_{e,t}\right)(1-\phi_t)dt + \rho_{e,t}(1-\chi)\frac{\underline{q}_t}{q_t}\phi_t dt - dc_t \tag{74}$$

We need to use Ito's Lemma including jumps to deal with the jump $df_t$ in the net worth and price evolution. Remember that if $df_t = 1$, $N_t$ jumps to zero, and $q_t$ jumps to $\underline{q}$. Ito's lemma gives:

$$d\eta_t = \frac{dN_t}{q_tK_t} - \frac{d(q_tK_t)N_t}{(q_tK_t)^2} - \frac{dN_t d(q_tK_t)}{(q_tK_t)^2} + \frac{N_t d(q_tK_t)^2}{(q_tK_t)^3} + \left(\frac{0}{\underline{q}K_t} - \frac{N_t}{q_tK_t}\right)df_t \tag{75}$$

Where it is understood that the jumps have been removed from $dN_t$ and $dq_t$. Rearranging gives:

$$\frac{d\eta_t}{\eta_t} = \frac{dN_t}{N_t} - \frac{d(q_tK_t)}{q_tK_t} - \frac{dN_t}{N_t}\frac{d(q_tK_t)}{q_tK_t} + \frac{d(q_tK_t)^2}{(q_tK_t)^2} - df_t \tag{76}$$

Using Ito's Lemma on $d(q_tK_t)$ and $d(q_tK_t)^2$gives:

$$\frac{d(q_tK_t)}{q_tK_t} = \frac{dq_t}{q_t} + \frac{dK_t}{K_t} + \frac{dq_t}{q_t}\frac{dK_t}{K_t} \tag{77}$$

$$\frac{d(q_tK_t)^2}{(q_tK_t)^2} = \frac{dq_t^2}{q_t^2} + \frac{dK_t^2}{K_t^2} + 2\frac{dq_t}{q_t}\frac{dK_t}{K_t} \tag{78}$$

Plugging this and the assumed $dq_t$ equation into the $d(q_tK_t)$ terms gives:

$$\frac{d(q_tK_t)}{q_tK_t} = (\Phi(\iota_t) - \delta\psi_t - \underline{\delta}(1-\psi_t) + \mu_t^q + \sigma\sigma_t^q)\,dt + (\sigma + \sigma_t^q)\,dZ_t \tag{79}$$

$$\frac{d(q_tK_t)^2}{(q_tK_t)^2} = (\sigma_t^q)^2dt + \sigma^2dt + 2\sigma\sigma_t^qdt = (\sigma + \sigma_t^q)^2dt \tag{80}$$

Now using definition of $dr_t^k$ to simplify a bit:

$$\frac{d(q_tK_t)}{q_tK_t} = dr_t^k - \frac{a-\iota_t}{q_t}dt - (1-\psi_t)(\underline{\delta} - \delta)dt \tag{81}$$

Calculating the cross term:

$$\frac{dN_t}{N_t}\frac{d(q_tK_t)}{q_tK_t} = \phi_t(\sigma + \sigma_t^q)^2dt \tag{82}$$

Putting all this together:

$$\frac{d\eta_t}{\eta_t} = (\phi_t - 1)(dr_t^k - \rho_h dt - \rho_{e,t}dt) + \rho_{e,t}(1-\chi)\frac{\underline{q}_t}{q_t}\phi_t dt + \frac{a-\iota_t}{q_t}dt + (1-\psi_t)(\underline{\delta} - \delta)dt$$

$$-(\phi_t - 1)(\sigma + \sigma_t^q)^2dt - dc_t - df_t \tag{83}$$

Which is exactly BrS' equation for the evolution of $\eta_t$, plus the extra $\rho_{e,t}$ terms. Equating terms with the guessed form for $d\eta_t$ in (19):

$$\sigma_t^\eta = (\phi_t - 1)(\sigma + \sigma_t^q) \tag{84}$$

$$\mu_t^\eta = (\phi_t - 1)(Edr_t^k - \rho_h - \rho_{e,t} - (\sigma + \sigma_t^q)^2) + \rho_{e,t}(1-\chi)\frac{\underline{q}_t}{q_t}\phi_t + \frac{a-\iota_t}{q_t} + (1-\psi_t)(\underline{\delta} - \delta) \tag{85}$$

Now if banks are holding positive leverage we can use their leverage FOC to simplify $\mu_t^\eta$ to:

$$\mu_t^\eta = -\sigma_t^\eta(\sigma + \sigma_t^q + \sigma_t^\theta) + \rho_{e,t}(1-\chi)\frac{\underline{q}}{q_t} + \frac{a-\iota_t}{q_t} + (1-\psi_t)(\underline{\delta} - \delta) \tag{86}$$

## B.5 Boundary conditions

The boundary conditions are identical to those in the baseline model of BrS' paper, and the interested reader is referred to their Proof of Proposition II.4.

# C Proofs

**Proof of Proposition 1**. Firstly note that $\frac{\partial(\sigma^\eta \eta)}{\partial \phi} > 0$, and thus $\frac{\partial \sigma^q}{\partial \phi} > 0$ and $\frac{\partial \sigma^\theta}{\partial \phi} < 0$ since $q' > 0$ and $\theta' < 0$ respectively. Now implicitly differentiate (26) with respect to $\rho_e$:

$$\phi'(\rho_e)\left[(\sigma^\theta)'(\sigma+\sigma^q) + (\sigma^q)'\sigma^\theta - \left(1-\frac{\underline{q}}{q}\right)\rho_e \chi \frac{\underline{q}}{q}\right] = \chi\frac{\underline{q}}{q}\left[1-\phi\left(1-\frac{\underline{q}}{q}\right)\right] \tag{87}$$

The term in square brackets on the right hand side is negative whenever crises are possible. The term in square brackets on the left hand side is negative because $\sigma^\theta$ and $(\sigma^\theta)'$ are negative, and $1-\underline{q}/q$ is positive whenever $\underline{q} < q$. □

**Proof of Proposition 4**. Since the household provides the insurance and is risk neutral, the premium for the insurance contract must satisfy $r_t^I = \rho_{e,t}$ so that the household breaks even.[18] Note that if the expert has an optimal plan that involves her going bankrupt during a crisis, then limited liability implies that she will never hold any of the insurance asset – it costs her in normal times and provides no benefits during a crisis. Thus the only way she might hold any is if it gives benefit in a plan where she will remain solvent during a crisis. In this region, and including the insurance asset, the expert's optimisation problem is now:

$$(\rho_b + \rho_{e,t})\theta_t = \max_{dc_t \geq 0, \phi_t \geq 0, \phi_t^I \geq 0} dc_t + ...$$

$$\mu_t^\theta \theta_t dt + ...$$

$$\left[\left(\frac{a-\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)\phi_t + \rho_h(1-\phi_t) - r_t^I\phi_t^I - \frac{dc_t}{dt}\right]\theta_t dt + ...$$

$$\sigma_t^\theta(\sigma+\sigma_t^q)\phi_t\theta_t dt + ...$$

$$\rho_{e,t}\underline{\theta}_t\left(1-\phi_t\left(1-\frac{\underline{q}}{q_t}\right)+\phi_t^I\right) \tag{88}$$

Where $k_t^I$ is the units of the insurance asset held, and $\phi_t^I = k_t^I/n_t$. Imposing that $r_t^I = \rho_{e,t}$, we see that the expert will optimally choose to hold zero insurance ($\phi_t^I = 0$) as long as $\underline{\theta}_t \leq \theta_t$, which is always satisfied if $\underline{\theta}_t = 1$ which is true in the baseline model (see Proposition 6). In this case insurance also doesn't affect the expert's choice of leverage or consumption relative to the case without insurance. □

[18] To see this, note that from $t$ to $t+dt$ the household earns the expected insurance premium $(1-\rho_{e,t}dt)r_t^I dt = r_t^I dt$, and has expected payout $\rho_{e,t}dt$. Setting expected profit to zero gives $r_t^I dt - \rho_{e,t}dt = 0 \Rightarrow r_t^I = \rho_{e,t}$.

# D Model solution with regulatory leverage constraint

This section outlines the solution of the model when experts face an exogenous borrowing constraint of the form $\phi_t \leq \bar{\phi}_t$. In the model this is interpreted as a regulatory leverage constraint, but the solution also applies to leverage constraints derived from limited commitment problems. We still conjecture that marginal value follows:

$$d\theta_t = \mu_t^\theta \theta_t dt + \sigma_t^\theta \theta_t dZ_t + df_t \left(\underline{\theta}_t - \theta_t\right) \tag{89}$$

Experts' value can be expressed as:

$$\rho_b \theta_t n_t = \max_{dC_t \geq 0, 0 \leq \phi_t \leq \bar{\phi}_t} \left\{dC_t + \mathrm{E}_t d(\theta_t n_t)\right\} \tag{90}$$

Using previous arguments we can show that:

$$(\rho_b + \rho_{e,t})\theta_t = \max_{dc_t \geq 0, 0 \leq \phi_t \leq \bar{\phi}_t} dc_t + ...$$

$$\mu_t^\theta \theta_t dt + ...$$

$$\left[\left(\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma \sigma_t^q\right) \phi_t + \left(\rho_h + \rho_{e,t}\right)(1 - \phi_t) + \rho_{e,t}(1 - \chi)\frac{\underline{q}_t}{q_t}\phi_t - \frac{dc_t}{dt}\right] \theta_t dt + ...$$

$$\sigma_t^\theta \left(\sigma + \sigma_t^q\right) \phi_t \theta_t dt$$

Where I have imposed that $\frac{\underline{n}_t}{n_t} = 0$, i.e. that the expert allows herself to go bankrupt during a crisis. The leverage first order condition gives:

$$\frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma \sigma_t^q - (\rho_h + \rho_{e,t}) + \rho_{e,t}(1 - \chi)\frac{\underline{q}_t}{q_t} + \sigma_t^\theta \left(\sigma + \sigma_t^q\right) = \lambda_t \geq 0 \tag{91}$$

Where $\lambda_t$ is the lagrange multiplier on the leverage constraint. As before, the first order condition for consumption gives:

$$\theta_t \geq 1 \tag{92}$$

with equality if $dc_t > 0$. At the optimum, and when $dc_t = 0$, evaluating the value function now gives:

$$\mu_t^\theta = \rho_b - \rho_h - \lambda_t \bar{\phi}_t \tag{93}$$

The rest of the model is the same, except we must be careful in the derivation of $\mu_t^\eta$ to use the new leverage first order condition.

# E Numerical solution and simulation

The algorithm to solve the model is based on BrS' original algorithm, which is detailed after their statement of Proposition II.4. The only difference is that I have additional terms in some of my

equations relating to the crisis price $\underline{q}$. In the baseline parameterisation with permanent crises this can be solved for at the beginning of the code, and passed as a parameter to the rest of the algorithm. Following BrS, the algorithm searches over values of $q'(0)$ to find the value which satisfies the required boundary conditions. Another difference from BrS is that I use a Newton-based algorithm to update my guesses for $q'(0)$ until I reach convergence, whereas they use a bisection algorithm.

The model is simulated by discretising time. For example, I simulate $\eta_t$ using its transition:

$$d\eta_t = \mu_t^\eta \eta_t dt + \sigma_t^\eta \eta_t dZ_t - \eta_t df_t$$

I choose a small value for $dt$, and draw values of $dZ_t$ from a normal distribution with mean zero and standard deviation $\sqrt{dt}$. Given $\eta_t$, the value at the next interval of time is found by the approximation $\eta_{t+dt} \simeq \eta_t + d\eta_t$. The jump process is approximated by a random variable which takes value one in every period with probability $1 - e^{-\rho_{e,t}dt}$ and zero otherwise. The model moments (average growth and standard deviations) are calculated from simulations of 5000 years, with $dt = 1/120$ (the results are unchanged by picking smaller values of $dt$). The expected time to experience a crisis is calculated by repeatedly simulating the economy, starting from $\eta^*$, and calculating how long it takes for the economy to experience a crisis, and averaging this over the trials.

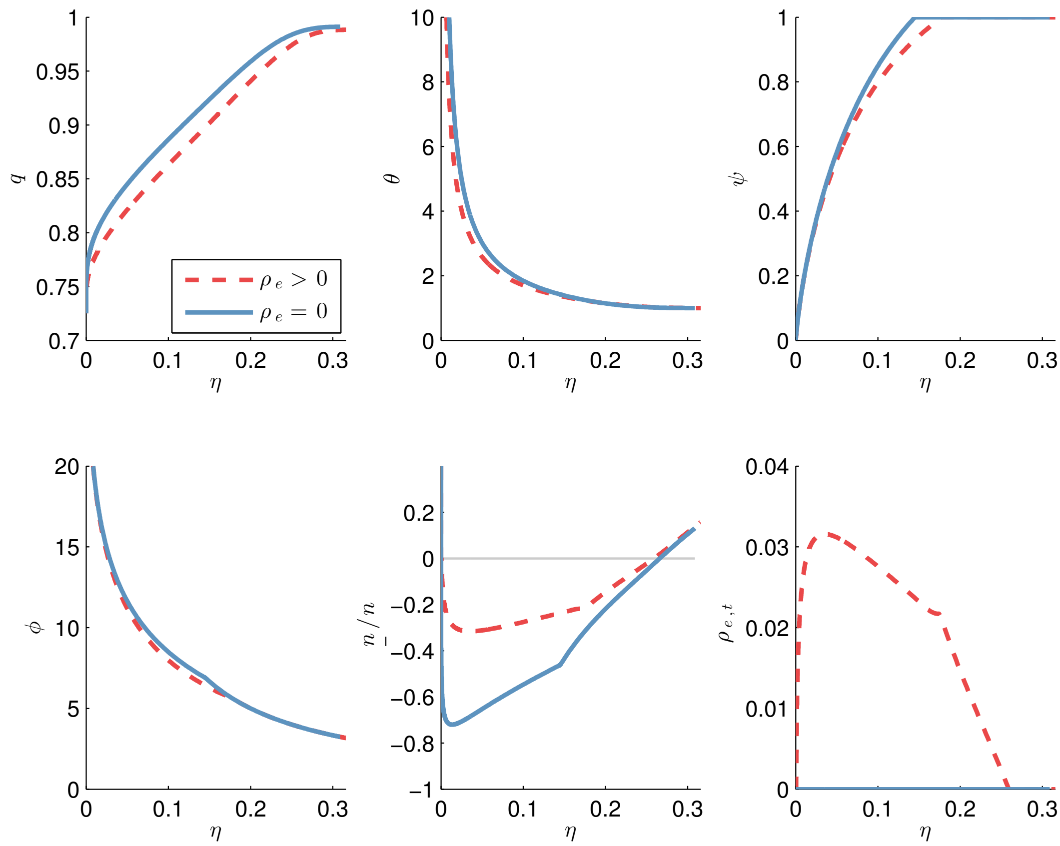
# F Model solution with $\chi > 0$

If I choose $\chi > 0$ then we can no longer use a constant value for the probability of coordinating on a crisis, as I did when $\chi = 0$. This is because this leads to discontinuous changes in leverage at the point where crises become possible. To deal with this, in this case I instead parameterise $\rho_{e,t}$ as:

$$\rho_{e,t} = \begin{cases} \rho_e |\frac{\underline{N}_t}{N_t}| & : \underline{N}_t < 0 \\ 0 & : \underline{N}_t \geq 0 \end{cases} \tag{94}$$

This parameterisation has appealing economic features, as well as being mathematically useful. Economically, it says that agents are more likely to coordinate on a crisis equilibrium the less well capitalised the banking sector is. Mathematically, this removes a discontinuity from the model around the point where a crisis just becomes possible. Conditional on this functional form, I have one free variable to choose which is the slope term $\rho_e$. The larger this parameter is the more likely we are to coordinate on the crisis equilibrium. Figure 9 plots the solution to the model in this case. The results are qualitatively similar to the results of the baseline model, except for leverage: leverage is now lower in some regions due to the experts' desire to deleverage to avoid paying the exogenous default costs.

Figure 9: Model solution with $\chi = 0.25$. Key variables

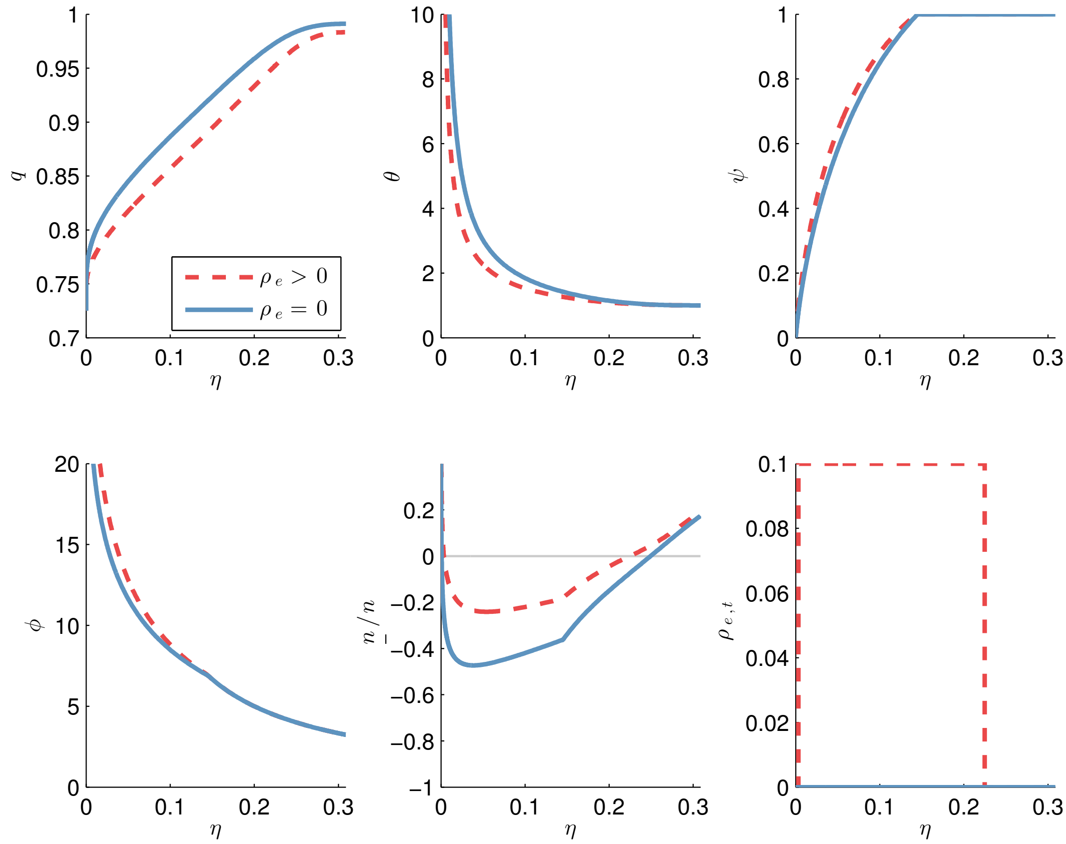



## G Model solution with non-permanent crises

In this section I solve the model with non permanent crisis. I keep the same parameters as the baseline model, including setting $\rho_e = 0.1$. I set the level of recapitalisation of the experts to $\hat{\eta} = 0.00005\eta^*$, and the flow intensity of recapitalisation to $\rho_r = 1.2427$. With these numbers, the chance of being recapitalised within one year is 71%, two years is 92%, and essentially 100% within around five years. Recapitalisation to that value of $\hat{\eta}$ implies that it takes roughly 15 years for the economy to naturally recover from $\hat{\eta}$ back to $\eta^*$, giving a total time from crisis to complete recovery of something around 20 years.

This generates a value of the crisis price of $\underline{q} = 0.783$, which is higher than the crisis price when crises are permanent (0.725). Since the crisis price is now slightly higher, crises are less likely, and the economies with and without crises spend 5% and 2.1% of their time in the crisis region under the stationary density (ignoring crisis realisations). Figure 10 plots key variables from the model solution. By comparison with Figure 3 it can be seen that the model solutions are qualitatively very similar.

Figure 10: Model solution non permanent crises




*The dashed red line gives the solution to the model with the baseline positive value of $\rho_e$, meaning the economy occasionally experiences crises. The solid blue line gives the solution where $\rho_e = 0$ and agents never coordinate on a crisis.*

# H Graphs and figures

Figure 11: Parameter sensitivity: Crisis net worth, $q$ and $\phi$ across changes in three parameters